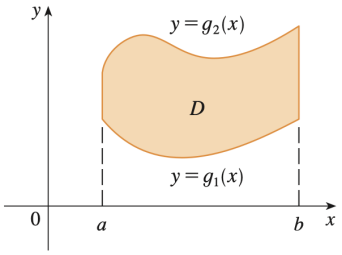
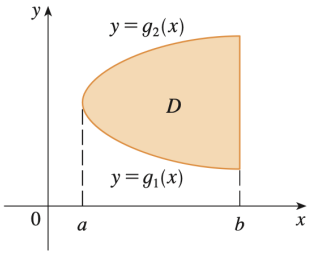
Lecture Notes
Math 2400 - Calculus III
Spring 2024

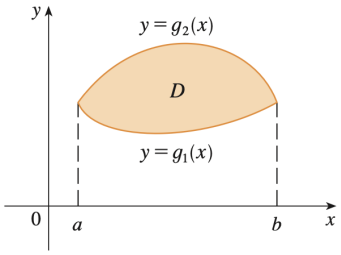
Name: _____

12.3 Double Integrals over General Regions

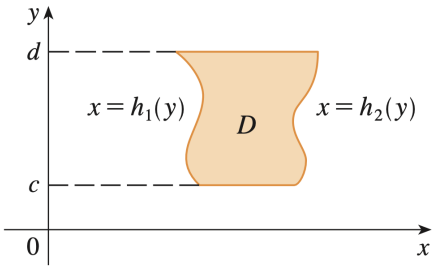
Definition. What is a type I region? If f is continuous on a type I region D, how can we evaluate the double integral of f over this region?

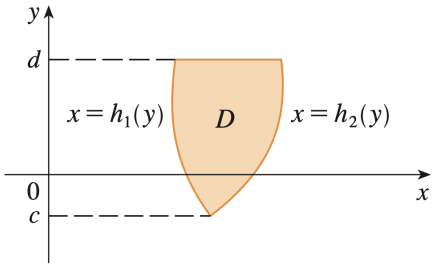




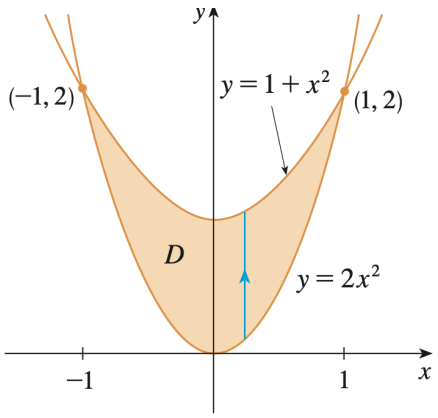


Definition. What is a type II region? If f is continuous on a type II region D, how can we evaluate the double integral of f over this region?

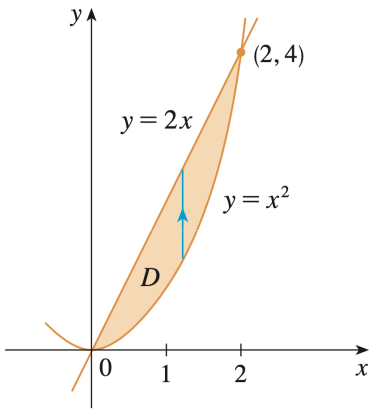


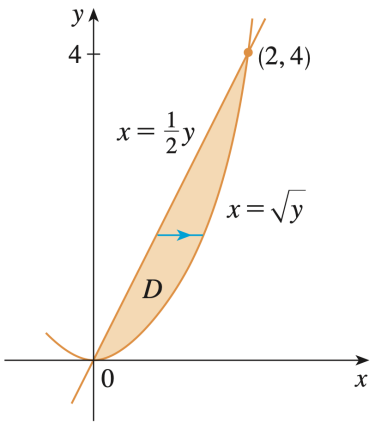


Example. Evaluate $\iint_D (x + 2y)\, dA$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

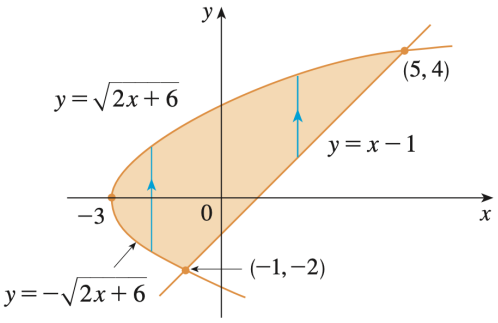


Example. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in the xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$.

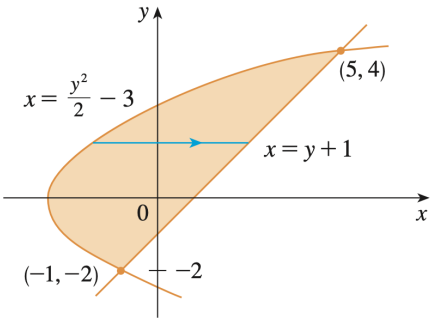




Example. Evaluate $\iint_D xy\,dA$, where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

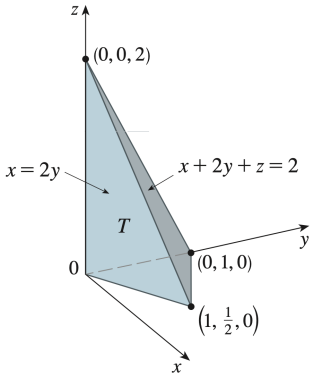


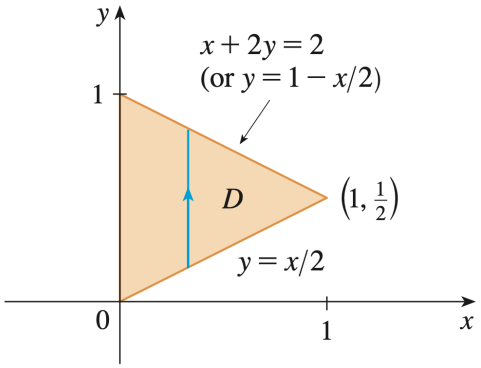
(a) D as a type I region



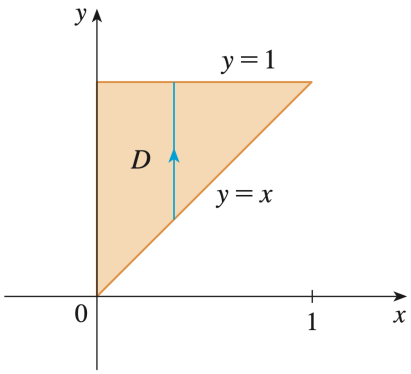
(b) D as a type II region

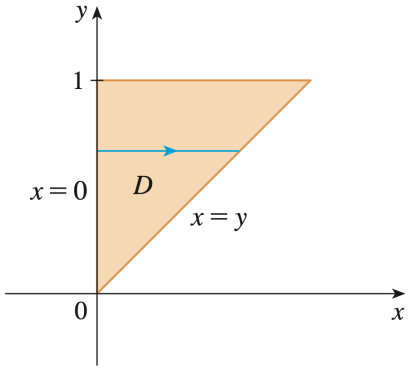
Example. Find the volume of the tetrahedron bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$, and $z = 0$.



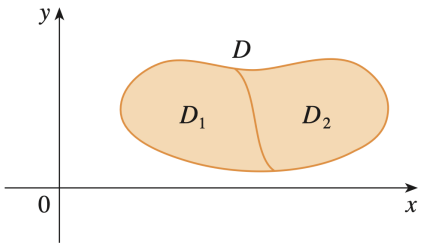


Example. Evaluate the iterated integral $\int_0^1 \int_x^1 \sin(y^2)\, dydx$.

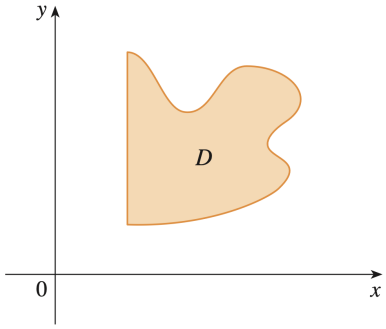




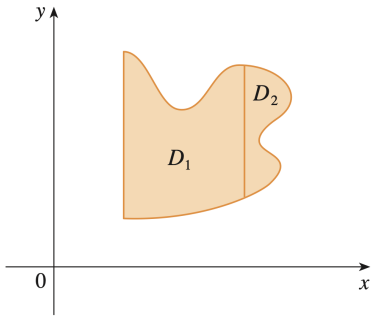
Question. Suppose $D = D_1 \cup D_2$, where D_1 and D_2 don't overlap except perhaps on their boundaries. How can we evaluate $\iint_D f(x, y)\, dA$?



Question. How can we use the above to evaluate double integrals over regions D that are neither type I nor type II?



(a) D is neither type I nor type II.



(b) $D = D_1 \cup D_2$; D_1 is type I, D_2 is type II.

Question. How can we use a double integral to compute the area of a region D?