Lecture Notes
Math 2400 - Calculus III
Spring 2024

Name: _____

12.2 Iterated Integrals

Question. How can we evaluate a double integral of a function $f(x, y)$ over a rectangle $R = [a, b] \times [c, d]$ by using iterated integrals?

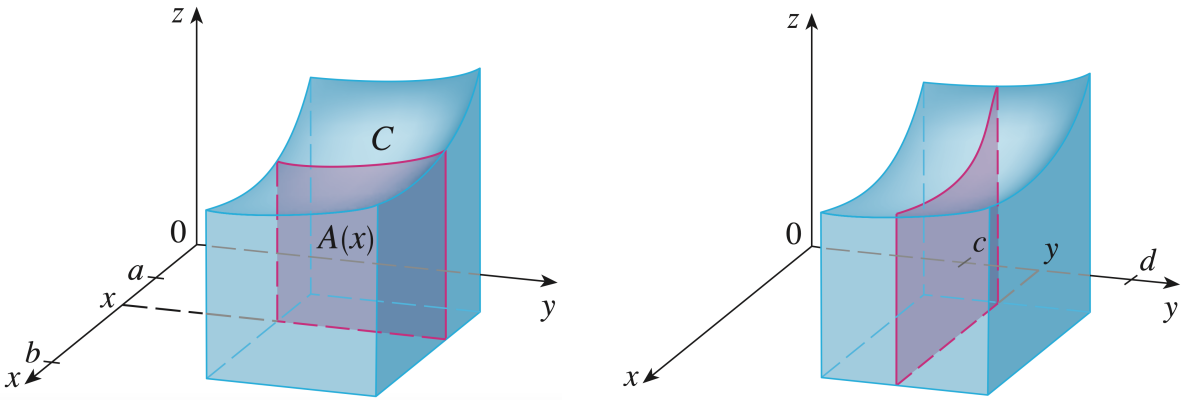
Example. Evaluate the iterated integrals.

(a) $\displaystyle\int_0^3 \int_1^2 x^2 y \, dy \, dx$

(b) $\displaystyle\int_1^2 \int_0^3 x^2 y \, dx \, dy$

Theorem (Fubini's Theorem)**.** If f is continuous on the rectangle $R = [a, b] \times [c, d]$, what does Fubini's Theorem say about $\iint_R f(x, y)\, dA$?

Proof.



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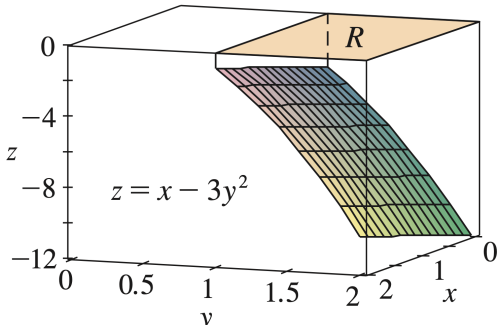
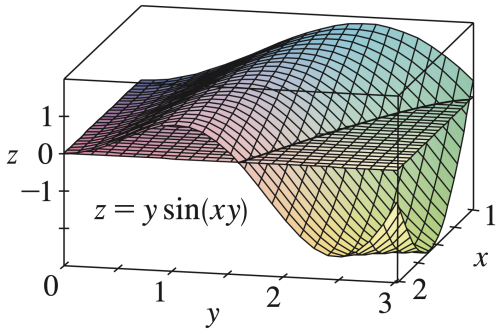
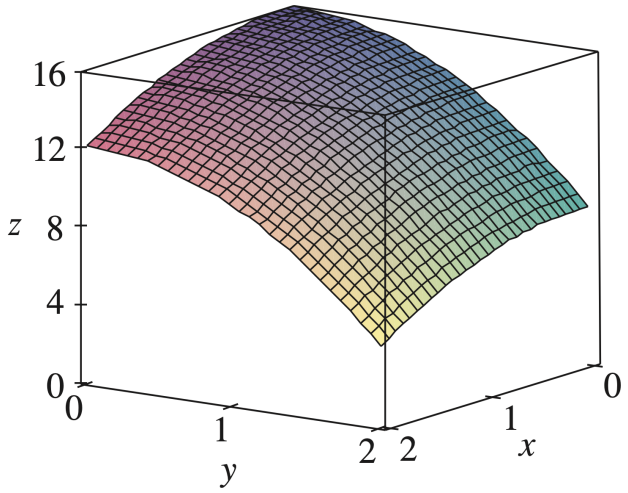
Example. Use Fubini's theorem to evaluate $\iint_R (x - 3y^2)\, dA$, where $R = [0, 2] \times [1, 2]$.



Example. Use Fubini's theorem to evaluate $\iint_R y \sin(xy)\, dA$ where $R = [1, 2] \times [0, \pi]$.



Example. Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$ and $y = 2$, and the three coordinate planes.



Theorem. If $f(x, y)$ can be factored as the product of a function of x only and a function of y only, how can we rewrite the double integral of f over a rectangle $R = [a, b] \times [c, d]$?

Proof.

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Example. Evaluate the integral $\iint_R \sin x \cos y \, dA$ if $R = [0, \pi/2] \times [0, \pi/2]$.