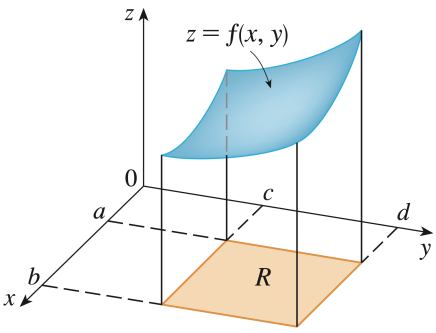
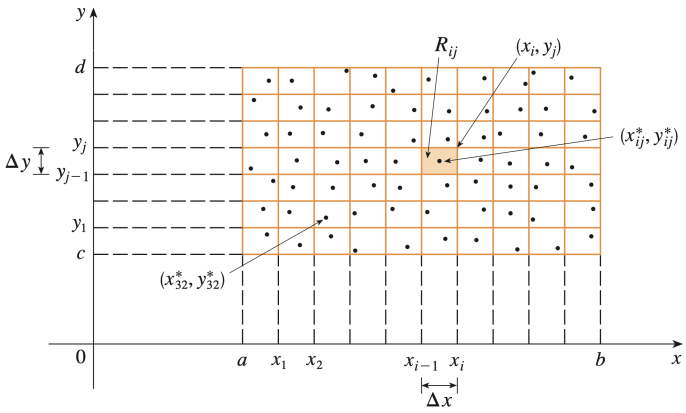
Lecture Notes
Math 2400 - Calculus III
Spring 2024

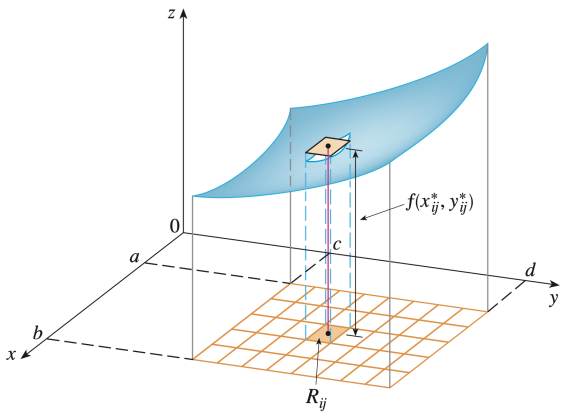
Name: _____

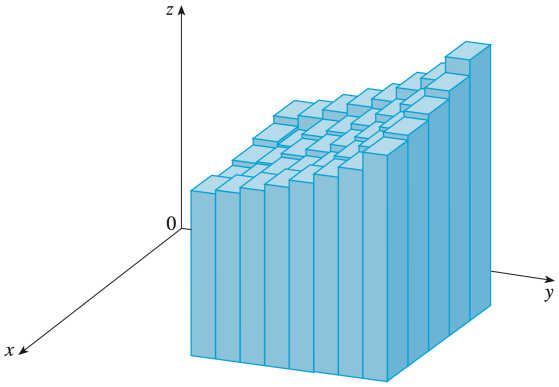
12.1 Double Integrals over Rectangles

Question. Consider a function f defined on a closed rectangle $R = [a, b] \times [c, d]$, and suppose that $f(x, y) \geq 0$. What is the volume of the solid S that lies above R and under the graph of f?



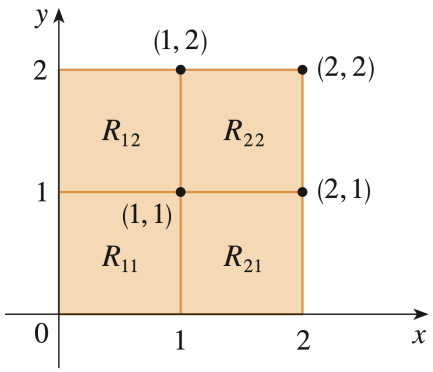


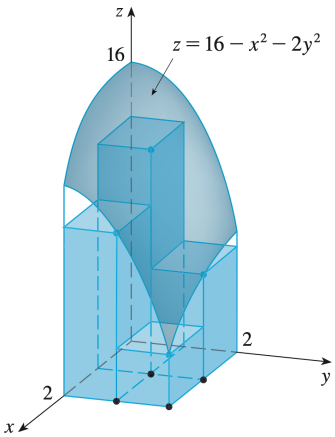


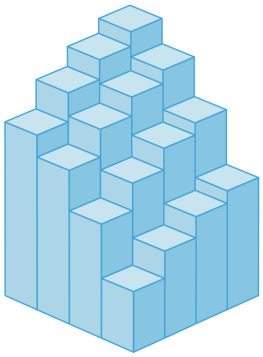


Definition. What is the double integral of f over the rectangle R?

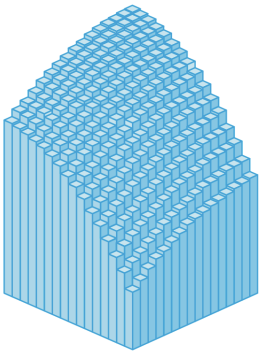
Example. Estimate the volume of the solid that lies above the square $R = [0, 2] \times [0, 2]$ and below the elliptic paraboloid $z = 16 - x^2 - 2y^2$. Divide R into four equal squares and choose the sample point to be the upper right corner of each square R_{ij}.







$m = n = 4, V \approx 41.5$

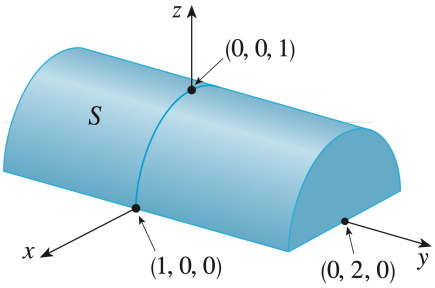


$m = n = 16, V \approx 46.46875$

Example. If $R = \{(x, y) \mid -1 \le x \le 1, -2 \le y \le 2\}$, evaluate the integral

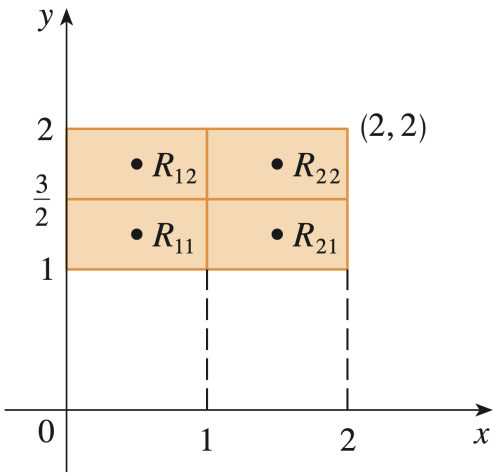
$$\iint_R \sqrt{1 - x^2}\, dA$$

by interpreting it geometrically.



Definition. What is the Midpoint Rule for double integrals?

Example. Use the Midpoint Rule with $m = n = 2$ to estimate the value of the integral $\iint_R (x - 3y^2)\, dA$, where $R = [0, 2] \times [1, 2]$.



Definition. What is the average value of a function $f(x, y)$ on a rectangle R?

Remark. Properties of Double Integrals

- $\iint_R [f(x, y) + g(x, y)]\, dA =$
- $\iint_R cf(x, y)\, dA =$
- What can we say if $f(x, y) \geq g(x, y)$ for all (x, y) in R?