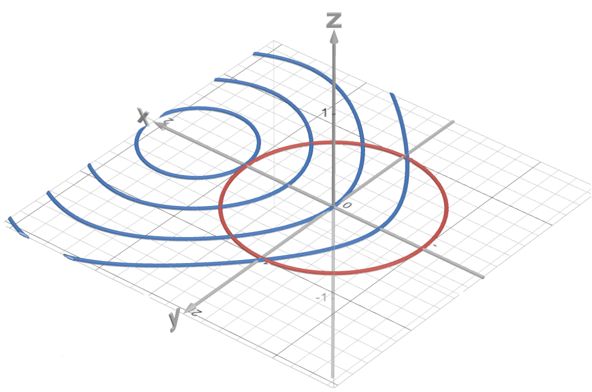
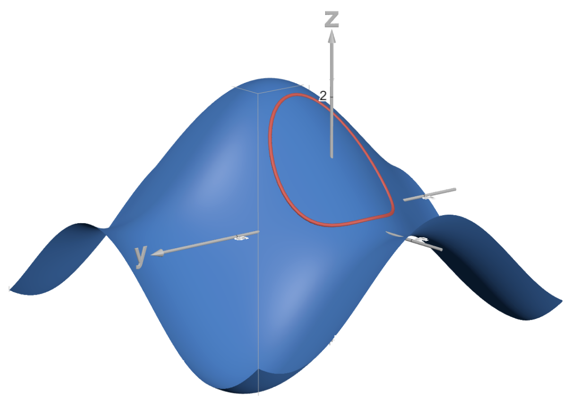
Lecture Notes
Math 2400 - Calculus III
Spring 2024

Name: ____________________________

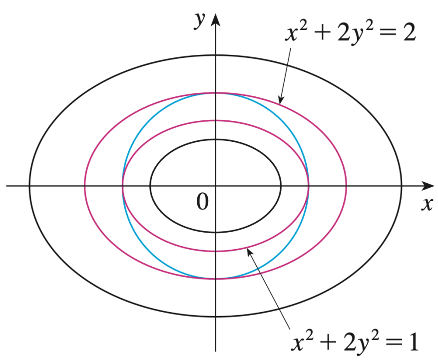
11.8 Lagrange Multipliers

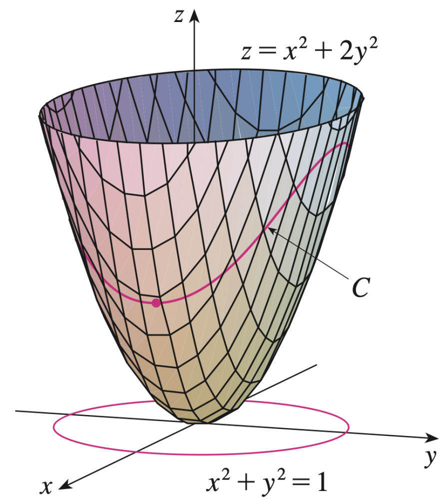
Question. What is the idea of Lagrange Multipliers?





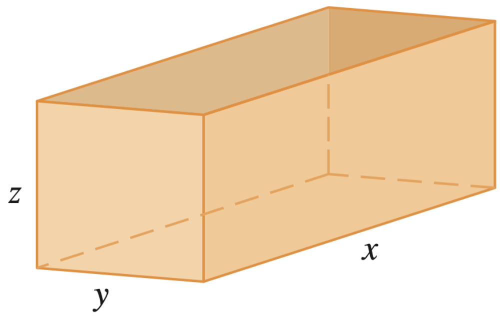
Example. Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.



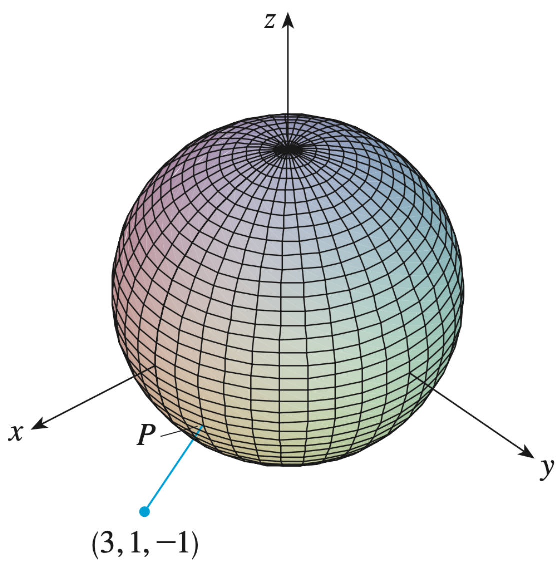


Example. Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the disk $x^2 + y^2 \leq 1$.

Example. A rectangular box without a lid is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box.



Example. Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3, 1, -1)$.



Method of Lagrange Multipliers

Theorem (Two Variables)**.** To find the maximum and minimum values of $f(x, y)$ subject to the constraint $g(x, y) = k$ (assuming that these extreme values exist and $\Delta g \neq 0$ on the curve $g(x, y) = k$):

(a) Find all values of x, y, and λ such that

$$\Delta f(x, y) = \lambda \Delta g(x, y)$$

and

$$g(x, y) = k$$

(b) Evaluate f at all the points x, y that result from step (a). The largest of these values is the maximum value of f; the smallest is the minimum value of f.

Writing the vector equation $\Delta f = \lambda \Delta g$ in terms of components, then the equations in step (a) become

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad g(x, y) = k$$

which is a system of three equations in the three unknowns x, y, and λ. It is not necessary to find explicit values for λ.

Theorem (Three Variables)**.** To find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$ (assuming that these extreme values exist and $\Delta g \neq 0$ on the surface $g(x, y, z) = k$):

(a) Find all values of x, y, z, and λ such that

$$\Delta f(x, y, z) = \lambda \Delta g(x, y, z)$$

and

$$g(x, y, z) = k$$

(b) Evaluate f at all the points x, y, z that result from step (a). The largest of these values is the maximum value of f; the smallest is the minimum value of f.

Writing the vector equation $\Delta f = \lambda \Delta g$ in terms of components, then the equations in step (a) become

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad f_z = \lambda g_z \qquad g(x, y, z) = k$$

which is a system of four equations in the four unknowns x, y, z, and λ. It is not necessary to find explicit values for λ.