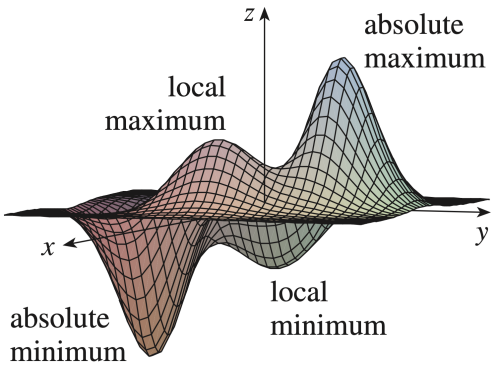
Lecture Notes
Math 2400 - Calculus III
Spring 2024

Name: _____

11.7 Maximum and Minimum Values

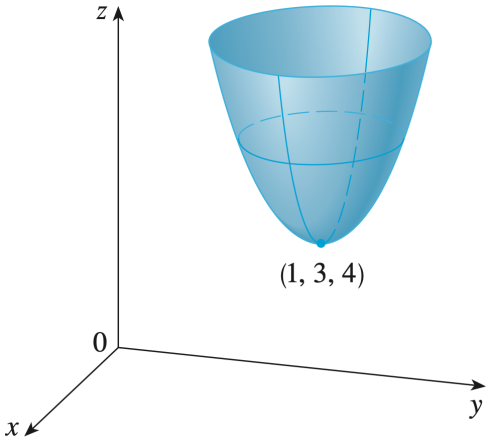
Question. What is a local maximum? Local minimum? Absolute maximum? Absolute minimum?



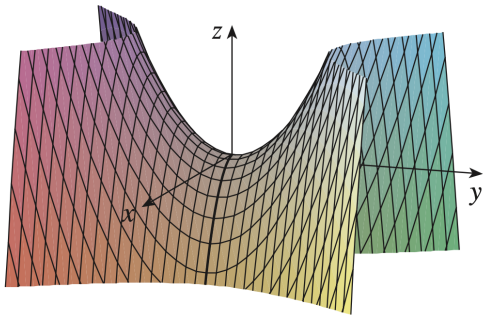
Theorem (Fermat's Theorem)**.** If f has a local maximum or minimum at (a, b), what can we say about the partial derivatives there?

Definition. What is a critical point?

Example. Find the extreme values of $f(x, y) = x^2 + y^2 - 2x - 6y + 14$.



Example. Find the extreme values of $f(x, y) = y^2 - x^2$.



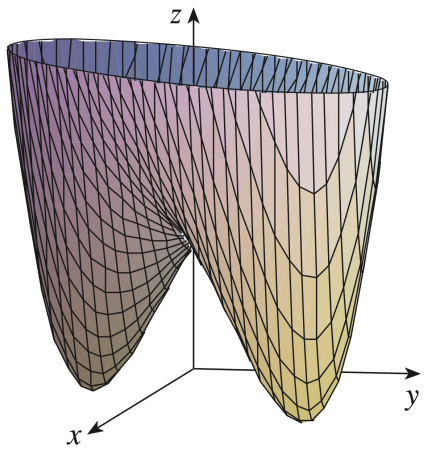
Theorem (Second Derivatives Test)**.** Define the Hessian matrix of $f(x, y)$ to be

$$H_f = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$$

How can we use the Hessian matrix to determine if f has a local maximum or local minimum at a point (a, b)?

Example. Find the local maximum and minimum values and saddle points of the function

$$f(x, y) = x^4 + y^4 - 4xy + 1$$

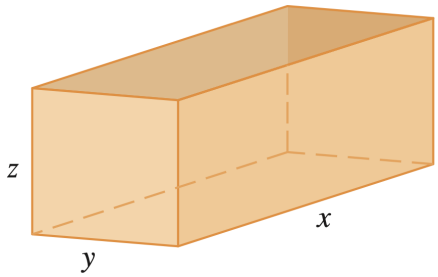


Example. Find and classify the critical points of the function

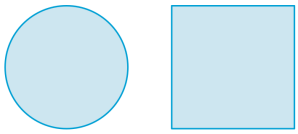
$$f(x, y) = 10x^2y - 5x^2 - 4y^2 - x^4 - 2y^4$$

Example. Find the shortest distance from the point $(1, 0, -2)$ to the plane $x + 2y + z = 4$.

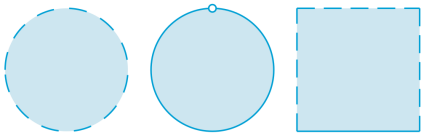
Example. A rectangular box without a lid is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box.



Definition. What is a closed set? What is a bounded set?



(a) Closed sets



(b) Sets that are not closed

Theorem (Extreme Value Theorem)**.** For a function f of one variable, the Extreme Value Theorem says that if f is continuous on a closed interval $[a, b]$, then f has an absolute minimum value and an absolute maximum value. What is the Extreme Value Theorem in two dimensions?

Example. Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \le x \le 3, 0 \le y \le 2\}$.