

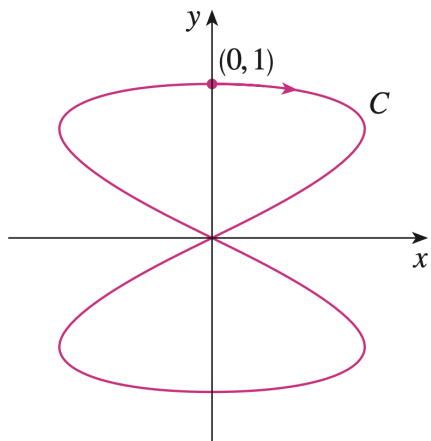
11.5 The Chain Rule

Question. For functions of more than one variable, the chain rule has several versions. What are the three different cases that we will be looking at?

Definition (Chain Rule: Case 1). Suppose that $z = f(x, y)$ is a differentiable function of x and y , where $x = g(t)$ and $y = h(t)$ are both differentiable functions of t . What is the chain rule in this case?

Example. If $z = x^2y + 3xy^4$, where $x = \sin 2t$ and $y = \cos t$, find $\frac{dz}{dt}$ when $t = 0$.

Question. In the above example, suppose that $z = T(x, y) = x^2y + 3xy^4$ represents the temperature at the point (x, y) . What does $\frac{dz}{dt}$ mean?



Definition (Chain Rule: Case 2). Suppose that $z = f(x, y)$ is a differentiable function of x and y , where $x = g(s, t)$ and $y = h(s, t)$ are differentiable functions of s and t . What is the chain rule in this case?

Example. If $z = e^x \sin y$, where $x = st^2$ and $y = s^2t$, find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

Definition (Chain Rule: General Version). Suppose that u is a differentiable function of the n variables $x_1, x_2, \dots, x_n$ and each x_j is a differentiable function of the m variables $t_1, t_2, \dots, t_m$. What is the chain rule in this case?

Example. Write out the chain rule for the case where $w = f(x, y, z, t)$ and $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$, and $t = t(u, v)$.

Example. If $u = x^4y + y^2z^3$, where $x = rse^t$, $y = rs^2e^{-t}$, and $z = r^2s \sin t$, find the value of $\frac{\partial u}{\partial s}$ when $r = 2, s = 1, t = 0$.

Definition. Suppose that an equation of the form $F(x, y) = 0$ defines y implicitly as a differentiable function of x . How can we use the chain rule to solve for $\frac{dy}{dx}$?

Example. Find y' if $x^3 + y^3 = 6xy$.

Definition. Suppose that an equation of the form $F(x, y, z) = 0$ defines z implicitly as a differentiable function of x and y . How can we use the chain rule to solve for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$?

Example. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $x^3 + y^3 + z^3 + 6xyz = 1$.