11.4 Tangent Planes and Linear Approximations

**Question.** Suppose a surface $S$ has equation $z = f(x, y)$, where $f$ has continuous first partial derivatives. What is the tangent plane to the surface $S$ at the point $P$?

**Question.** Suppose $f$ has continuous partial derivatives. What is an equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$? Relate this to the equation of a tangent line of a function $f(x)$. 
Example. Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.

Definition. What is the linearization of $f(x, y)$ at $(a, b)$? What is the the linear approximation of $f(x, y)$ at $(a, b)$?
Example. Find the linearization of $f(x, y) = xe^{xy}$ at $(1, 0)$. Then use it to approximate $f(1.1, -0.1)$. 
Example. How can we find the tangent plane to a parametric surface $S$ traced out by a vector function

$$\vec{r}(u, v) = x(u, v)\hat{i} + y(u, v)\hat{j} + z(u, v)\hat{k}$$

at a point $P_0$ with position vector $\vec{r}(u_0, v_0)$?
Example. Find the tangent plane to the surface with parametric equations $x = u^2$, $y = v^2$, $z = u + 2v$ at the point $(1,1,3)$. 