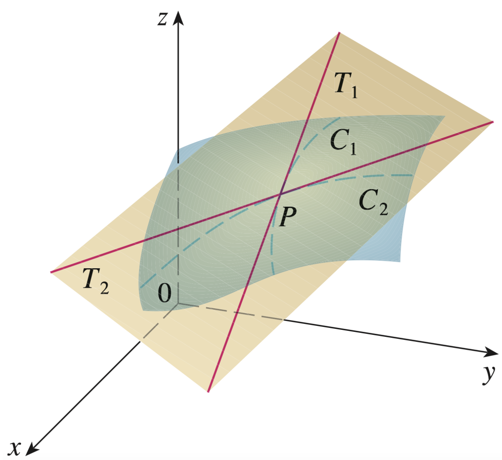
Lecture Notes
Math 2400 - Calculus III
Spring 2024

Name: ____________________________

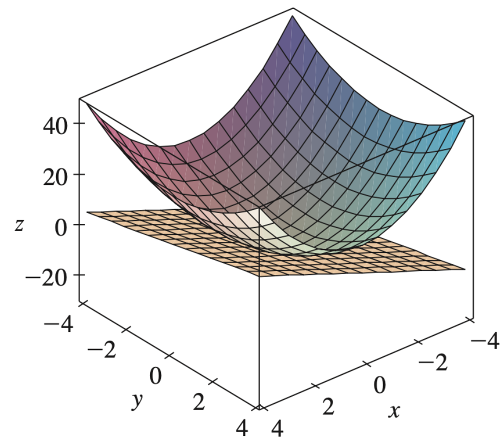
11.4 Tangent Planes and Linear Approximations

Question. Suppose a surface S has equation $z = f(x, y)$, where f has continuous first partial derivatives. What is the tangent plane to the surface S at the point P?



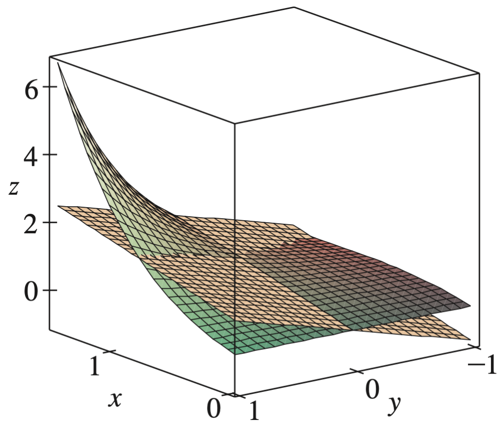
Question. Suppose f has continuous partial derivatives. What is an equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$? Relate this to the equation of a tangent line of a function $f(x)$.

Example. Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.



Definition. What is the linearization of $f(x, y)$ at (a, b)? What is the the linear approximation of $f(x, y)$ at (a, b)?

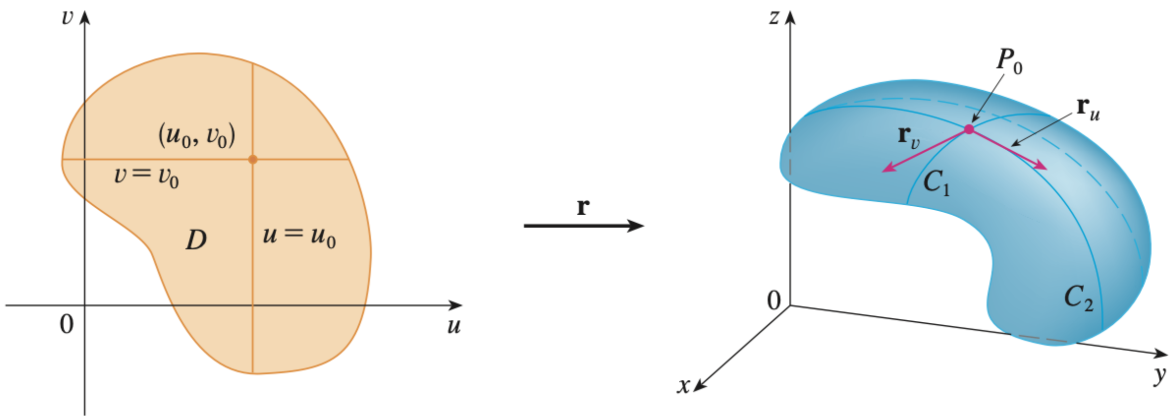
Example. Find the linearization of $f(x, y) = xe^{xy}$ at $(1, 0)$. Then use it to approximate $f(1.1, -0.1)$.



Example. How can we find the tangent plane to a parametric surface S traced out by a vector function

$$\vec{r}(u,v) = x(u,v)\vec{i} + y(u,v)\vec{j} + z(u,v)\vec{k}$$

at a point P_0 with position vector $\vec{r}(u_0,v_0)$?



Example. Find the tangent plane to the surface with parametric equations $x = u^2$, $y = v^2$, $z = u + 2v$ at the point $(1, 1, 3)$.