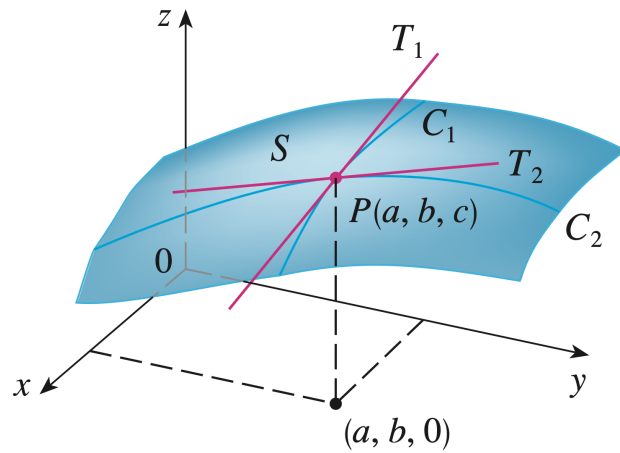


11.3 Partial Derivatives

Question. What is a partial derivative geometrically?



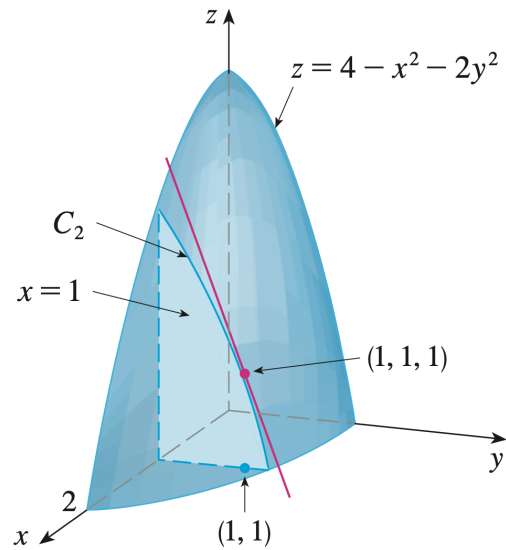
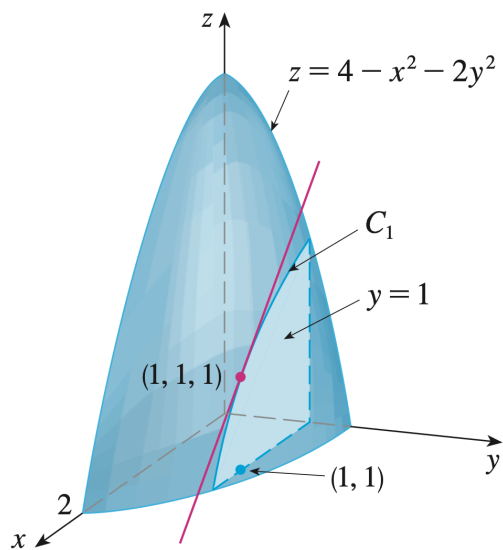
Definition. What are the partial derivatives of $f(x, y)$?

Remark. How to compute the partial derivatives of $z = f(x, y)$?

Remark. If $z = f(x, y)$, what are various notations for the partial derivatives?

Example. If $f(x, y) = x^3 + x^2y^3 - 2y^2$, find $f_x(2, 1)$ and $f_y(2, 1)$.

Example. If $f(x, y) = 4 - x^2 - 2y^2$, find $f_x(1, 1)$ and $f_y(1, 1)$ and interpret these numbers as slopes.



Example. If $f(x, y) = \sin\left(\frac{x}{1+y}\right)$, calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

Example. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by the equation

$$x^3 + y^3 + z^3 + 6xyz = 1$$

Definition. In general, if u is a function of n variables, $u = f(x_1, x_2, \dots, x_n)$, what is the partial derivative with respect to the i th variable x_i ?

Example. Find f_x , f_y , and f_z if $f(x, y, z) = e^{xy} \ln(z)$.

Definition. What are the second partial derivatives of $z = f(x, y)$?

Example. Find the second partial derivatives of $f(x, y) = x^3 + x^2y^3 - 2y^2$.

Theorem. What does Clairaut's Theorem say about mixed partials?

Definition. What is Laplace's equation? What are solutions to this equation called?

Example. Show that the function $u(x, y) = e^x \sin y$ is a solution of Laplace's equation.

Definition. What is the wave equation? What does this describe?

Example. Verify that the function $u(x, t) = \sin(x - at)$ satisfies the wave equation.