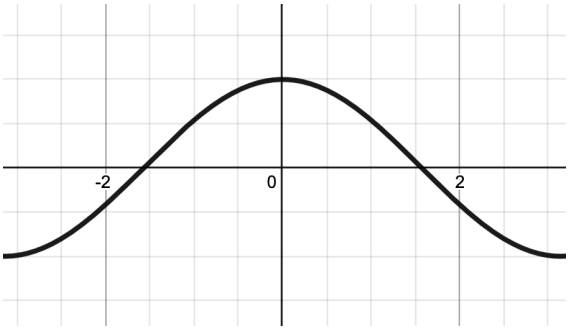
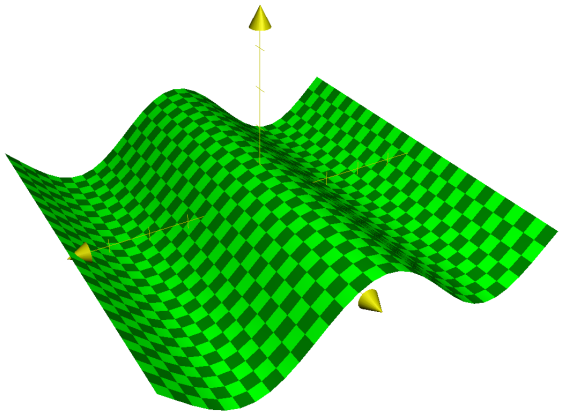
Lecture Notes
Math 2400 - Calculus III
Spring 2024

Name: _____

11.2 Limits and Continuity

Question. How is evaluating limits of functions in three dimensions different from evaluating limits in two dimensions?

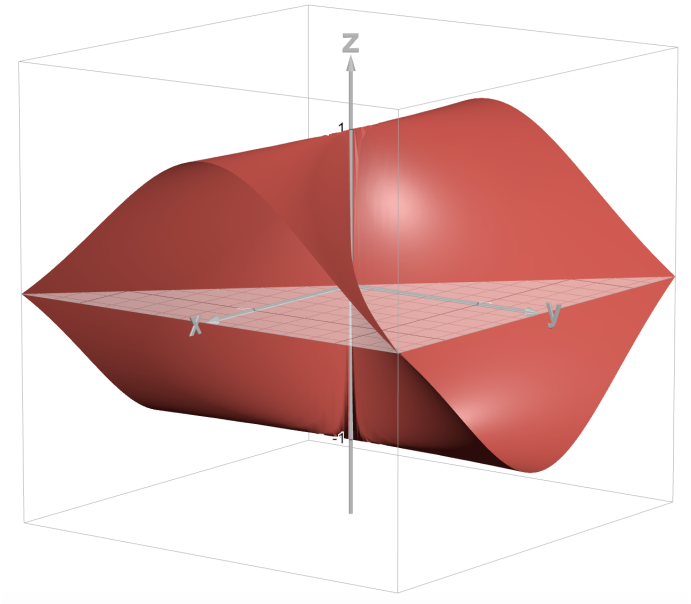




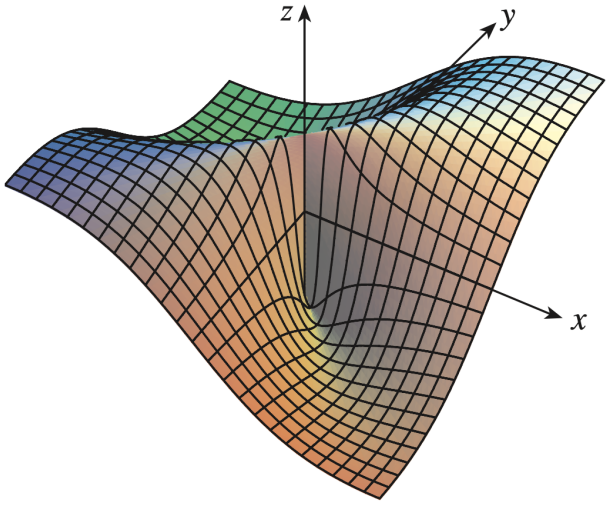
Definition. What does it mean for the limit of $f(x, y)$ as (x, y) approaches (a, b) to be L?

Question. How can we easily show that $\lim_{(x,y)\to(a,b)} f(x,y)$ does not exist?

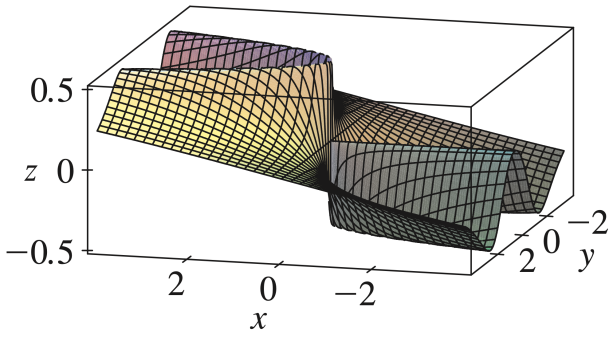
Example. Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$ does not exist.



Example. Does $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ exist?



Example. Does $\lim_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2+y^4}$ exist?



Example. Find $\lim_{(x,y)\to(0,0)} \dfrac{3x^2y}{x^2+y^2}$ if it exists.

Definition. What does it mean for a function $f(x, y)$ to be continuous at (a, b). How to evaluate limits of continuous functions?

Example. What are some examples of continuous functions?

Example. Evaluate $\lim_{(x,y)\to(1,2)} x^2y^2 - x^3y^2 + 3x + 2y$.

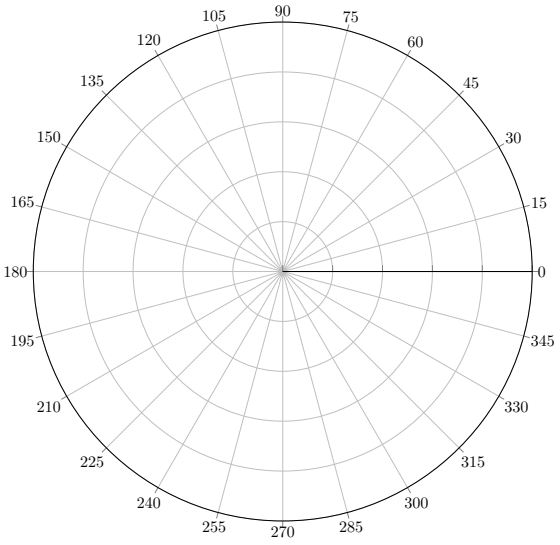
Example. Where is the function $f(x, y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ continuous?

Example. How can we extend things to functions of three or more variables?

Example. Where is the function $f(x, y, z) = \dfrac{\sqrt{y}}{x^2 - y^2 + z^2}$ continuous?

Example. Using the squeeze theorem, we showed that $\lim_{(x,y)\to(0,0)} \dfrac{3x^2y}{x^2+y^2} = 0$. Use polar coordinates to again show that this limit is 0. What does the squeeze theorem look like in this case?

Example. We showed that $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ does not exist by showing that $f(x,y) \to 0$ along the x-axis, but $f(x,y) \to \frac{1}{2}$ along the line $y = x$. What does this look like using polar coordinates?



Example. Find $\lim_{(x,y,z)\to(0,0,0)} \dfrac{xyz}{x^2+y^2+z^2}$ if it exists.

Example. Find $\lim_{(x,y,z)\to(0,0,0)} \dfrac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$ if it exists.