

11.1 Functions of Several Variables

Question. What are some ways to represent a function of two or more variables?

Example. The wind-chill index is often used to describe the apparent severity of the cold. This index W is a subjective temperature that depends on the actual temperature T and the wind speed v . So W is a function of T and v , and we can write $W = f(T, v)$. The table below records values of W compiled by the National Weather Service of the US and the Meteorological Service of Canada.

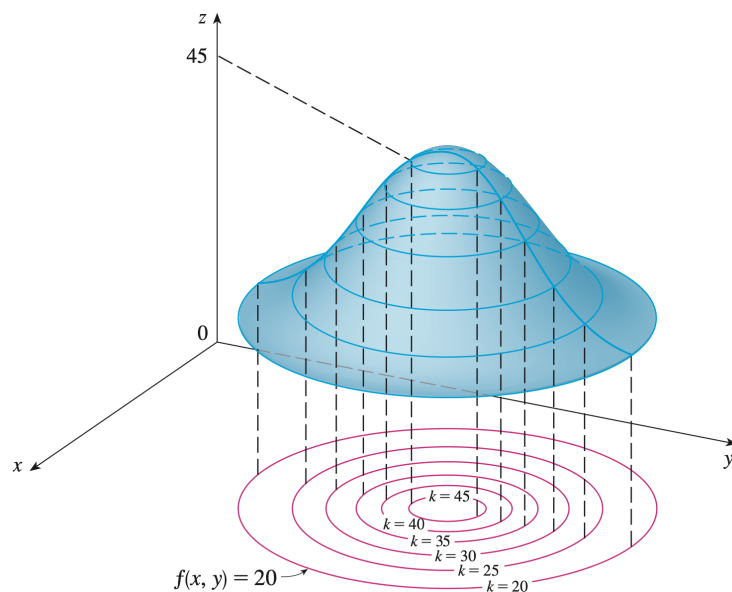
		Wind speed (km/h)										
Actual temperature (°C)	$T \backslash v$	5	10	15	20	25	30	40	50	60	70	80
	5	4	3	2	1	1	0	−1	−1	−2	−2	−3
	0	−2	−3	−4	−5	−6	−6	−7	−8	−9	−9	−10
	−5	−7	−9	−11	−12	−12	−13	−14	−15	−16	−16	−17
	−10	−13	−15	−17	−18	−19	−20	−21	−22	−23	−23	−24
	−15	−19	−21	−23	−24	−25	−26	−27	−29	−30	−30	−31
	−20	−24	−27	−29	−30	−32	−33	−34	−35	−36	−37	−38
	−25	−30	−33	−35	−37	−38	−39	−41	−42	−43	−44	−45
	−30	−36	−39	−41	−43	−44	−46	−48	−49	−50	−51	−52
	−35	−41	−45	−48	−49	−51	−52	−54	−56	−57	−58	−60
	−40	−47	−51	−54	−56	−57	−59	−61	−63	−64	−65	−67

Example. Find the domain and range of $g(x, y) = \sqrt{9 - x^2 - y^2}$

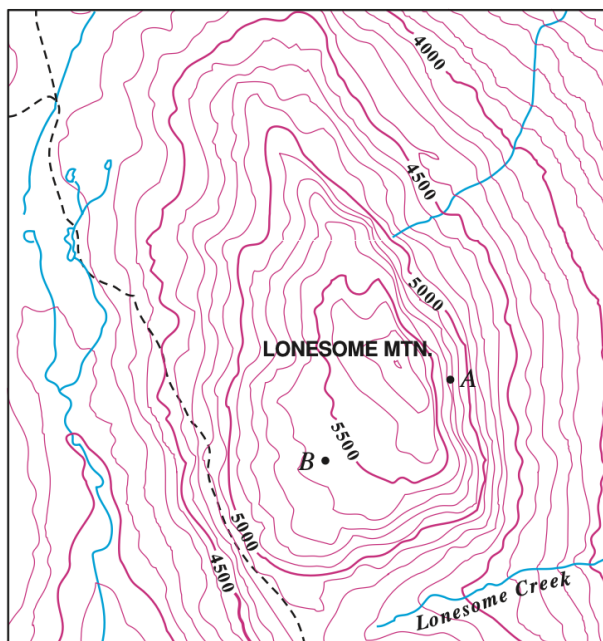
Example. Sketch the graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$.

Remark. Can an entire sphere be represented by a single function of x and y ?

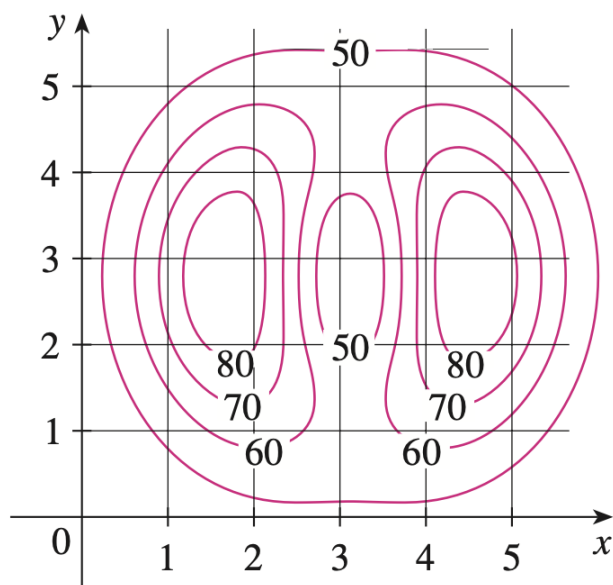
Definition. What are the level curves of a function f of two variables?



Example. What is a common example of level curves?



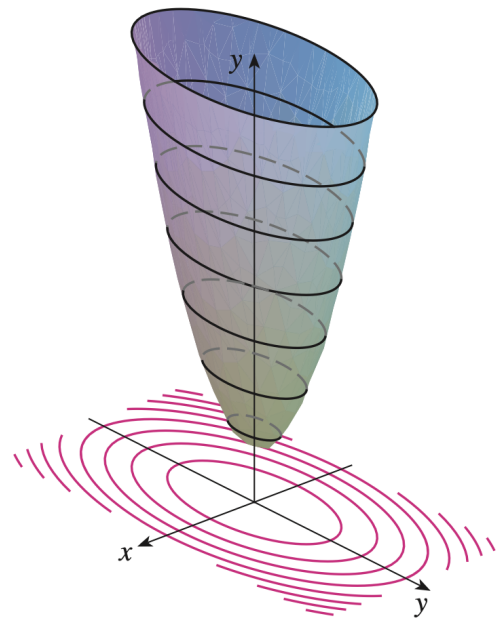
Example. A contour map for a function f is shown below. Use it to estimate the values of $f(1, 3)$ and $f(4, 5)$.



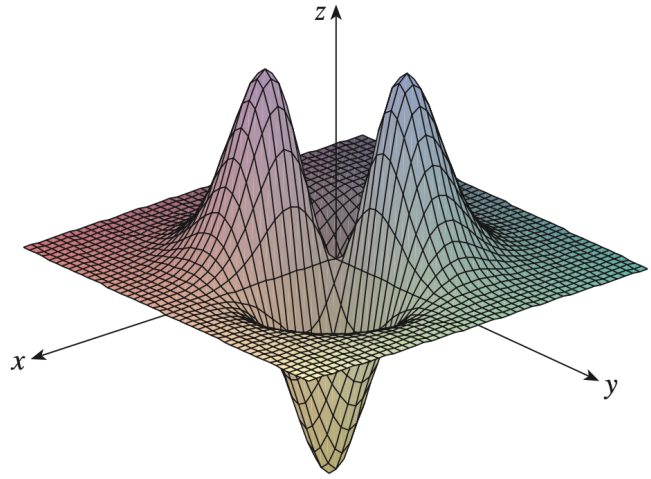
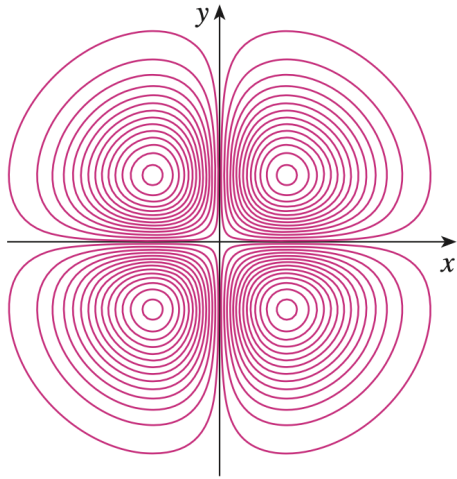
Example. Sketch the level curves of the function $f(x, y) = 6 - 3x - 2y$ for the values $k = -6, 0, 6, 12$.

Example. Sketch the level curves of the function $g(x, y) = \sqrt{9 - x^2 - y^2}$ for $k = 0, 1, 2, 3$.

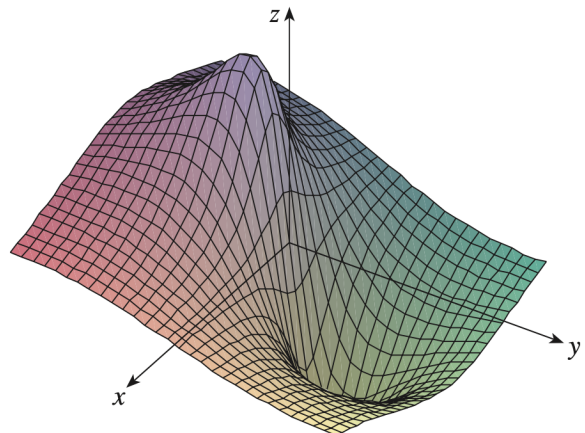
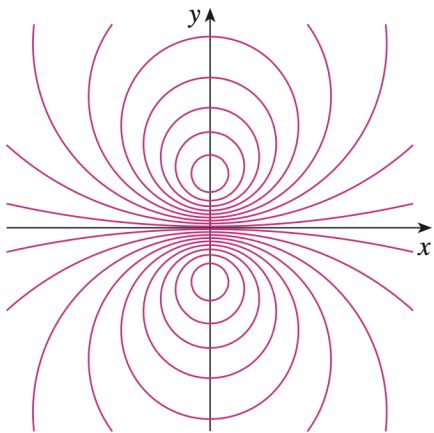
Example. Sketch some level curves of the function $h(x, y) = 4x^2 + y^2 + 1$.



Example. Examine the level curves of the function $f(x, y) = -xye^{-x^2-y^2}$.



Example. Examine the level curves of the function $f(x, y) = \frac{-3y}{x^2 + y^2 + 1}$.

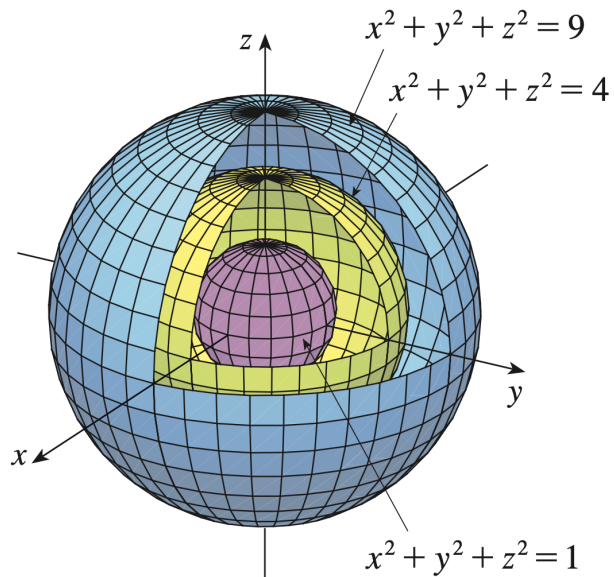


Definition. What is a function of three variables? Give an example.

Example. Find the domain of $f(x, y, z) = \ln(z - y) + xy \sin z$.

Definition. What are the level surfaces of a function f of three variables?

Example. Find the level surfaces of the function $f(x, y, z) = x^2 + y^2 + z^2$.



Definition. What is a function of n variables? Give an example.

Remark. How can we use vector notation to write a function of n variables more compactly?