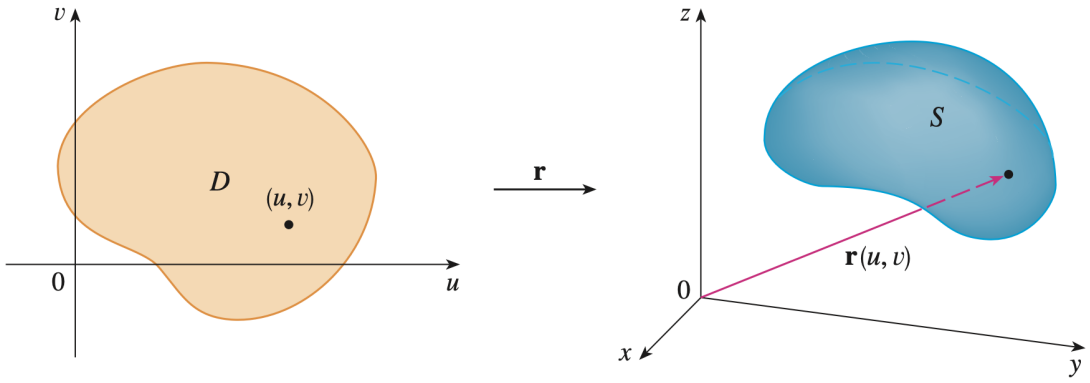
Lecture Notes
Math 2400 - Calculus III
Spring 2024

Name: _____

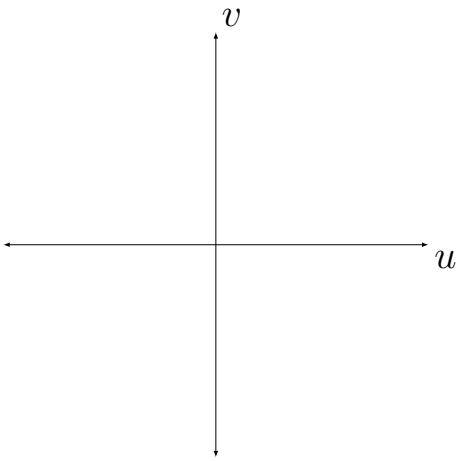
10.5 Parametric Surfaces

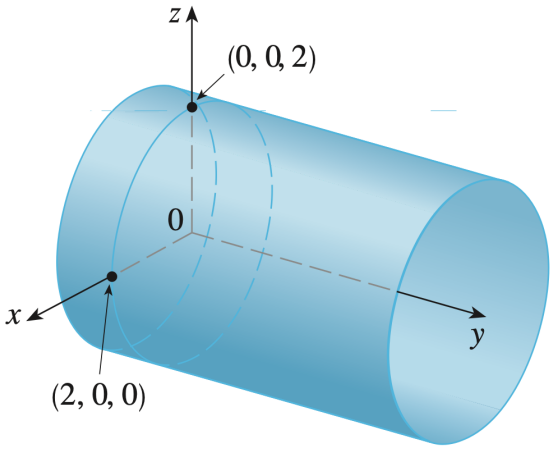
Definition. What is a parametric surface?



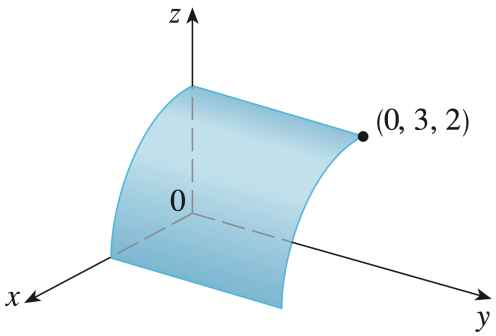
Example. Identify and sketch the surface with vector equation

$$\vec{r}(u, v) = 2\cos u\vec{i} + v\vec{j} + 2\sin u\vec{k}$$

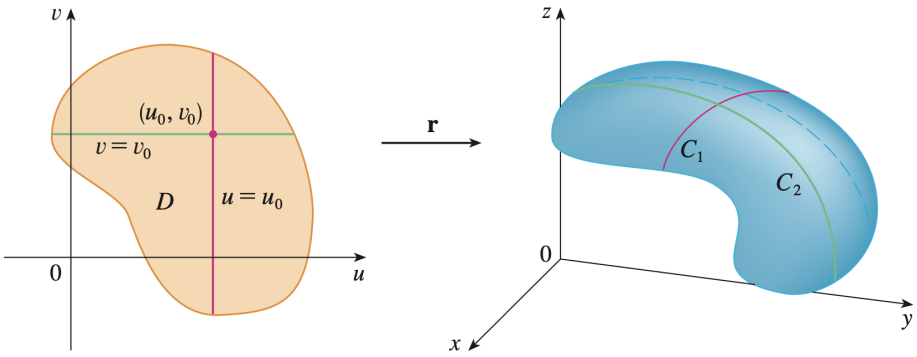




Question. How can we modify the previous example to obtain a quarter-cylinder with length 3?



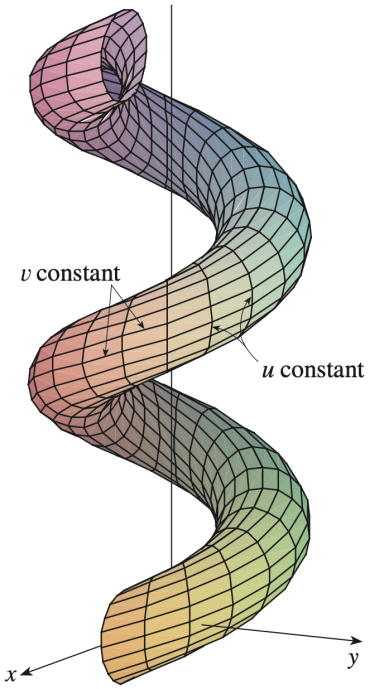
Definition. What are grid curves?



Example. The surface

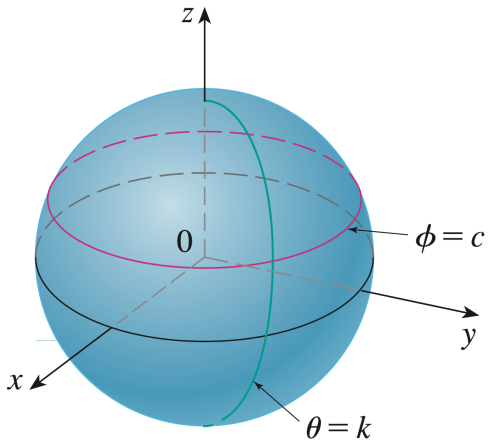
$$\vec{r}(u, v) = \langle (2 + \sin v) \cos u, (2 + \sin v) \sin u, u + \cos v \rangle$$

is graphed below. Which grid curves have u constant? Which have v constant?



Example. Find a vector function that represents the plane that passes through the point P_0 with position vector r_0 and that contains two non-parallel vectors $\vec{a}$ and $\vec{b}$.

Example. Find a parametric representation of the sphere $x^2 + y^2 + z^2 = a^2$. What are the grid curves?



Example. Find a parametric representation for the cylinder

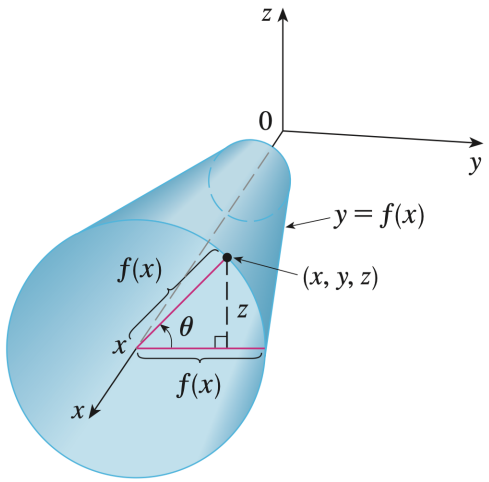
$$x^2 + y^2 = 4 \qquad 0 \leq z \leq 1$$

Remark. Suppose a surface S is given as the graph of a function of x and y, that is, with an equation of the form $z = f(x, y)$. How can we view S as a parametric surface?

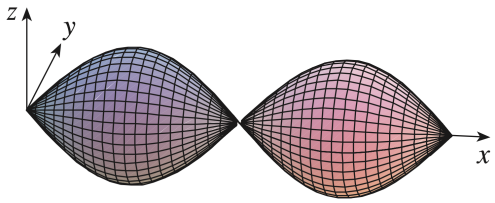
Example. Find a vector function that represents the elliptic paraboloid $z = x^2 + 2y^2$.

Example. Find a parametric representation for the top half of the cone $z^2 = 4x^2 + 4y^2$.

Definition. What is a surface of revolution? How can we represent a surface of revolution parametrically?



Example. Find parametric equations for the surface generated by rotating the curve $y = \sin x, 0 \leq x \leq 2\pi$ about the x-axis. Use these equations to graph the surface of revolution.



Example. Find a parametric representation for the cone $x^2 = 4(y^2 + z^2)$.