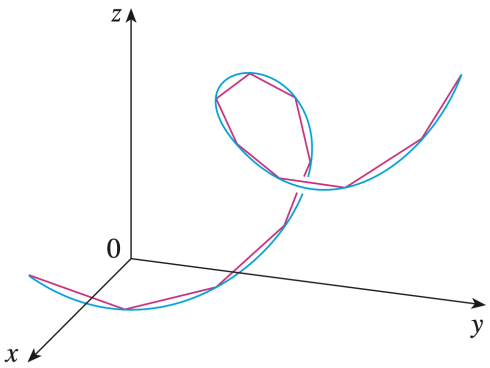
Lecture Notes
Math 2400 - Calculus III
Spring 2024

Name: _____

10.3 Arc Length

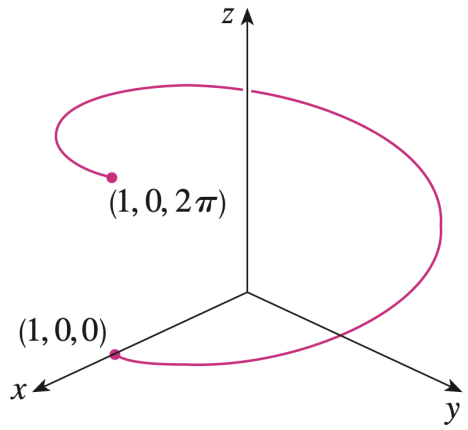
Question. How do we find the length of a plane curve with parametric equations $x = f(t), y = g(t)$ for $a \leq t \leq b$?

Definition. How do we find the length of a space curve?



Remark. How can we write the arc length formula more compactly?

Example. Find the length of the arc of the circular helix with vector equation $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + t\vec{k}$ from the point $(1, 0, 0)$ to the point $(1, 0, 2\pi)$.

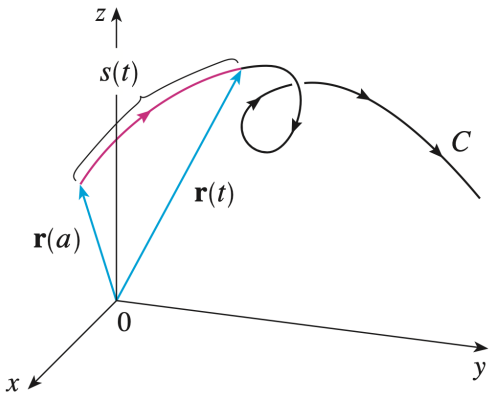


Definition. Can a single curve C be represented by more than one vector function? What is a parametrization? Does arc length depend on the parametrization of C?

Definition. Suppose that C is a curve given by a vector function

$$\vec{r}(t) = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k} \qquad a \le t \le b$$

where $\vec{r}\,'$ is continuous and C is traversed exactly once as t increases from a to b. What is the arc length function $s(t)$?



Remark. Why is it often useful to parametrize a curve with respect to arc length?

Example. Reparametrize the helix $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + t\vec{k}$ with respect to arc length measured from $(1, 0, 0)$ in the direction of increasing t.