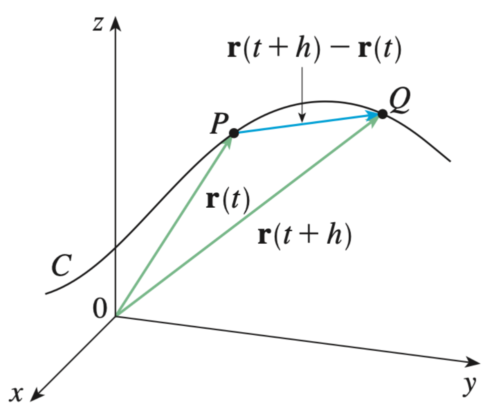
Lecture Notes
Math 2400 - Calculus III
Spring 2024

Name: ____________________________

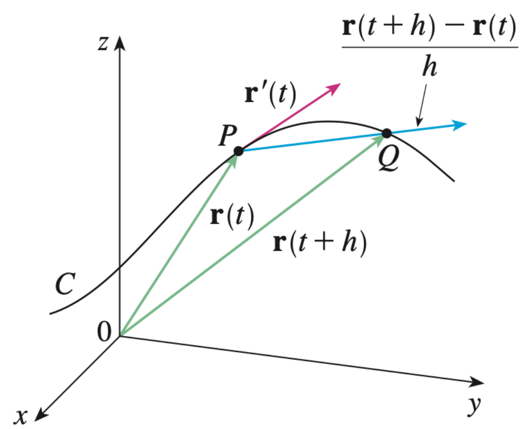
10.2 Derivatives and Integrals of Vector Functions

Definition. What is the derivative of a vector function $\vec{r}(t)$?

Question. Explain the derivative of a vector function geometrically.



(a) The secant vector



(b) The tangent vector

Definition. What is the unit tangent vector?

Theorem. If $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$, where f, g, and h are differentiable functions, show that $\vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle$.

Example.

(a) Find the derivative of $\vec{r}(t) = (1+t^3)\vec{i} + te^{-t}\vec{j} + \sin 2t\vec{k}$.

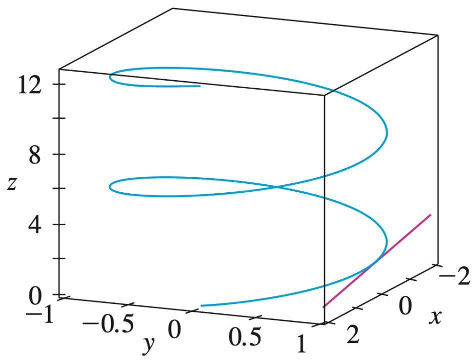
(b) Find the unit tangent vector at the point where $t = 0$.

Example. For the curve $\vec{r}(t) = \sqrt{t}\vec{i} + (2-t)\vec{j}$, find $\vec{r}\,'(t)$ and sketch the position vector $\vec{r}(1)$ and the tangent vector $\vec{r}\,'(1)$.

Example. Find parametric equations for the tangent line to the helix with parametric equations

$$x = 2\cos t \qquad y = \sin t \qquad z = t$$

at the point $(0, 1, \pi/2)$.



Definition. What is the second derivative of a vector function $\vec{r}$? What is the second derivative of the function in the previous example?

Theorem. Suppose $\vec{u}$ and $\vec{v}$ are differentiable vector functions, c is a scalar, and f is a real-valued function. Find differentation formulas for the following expressions.

1. $\dfrac{d}{dt}[\vec{u}(t) + \vec{v}(t)] =$

2. $\dfrac{d}{dt}[c\vec{u}(t)] =$

3. $\dfrac{d}{dt}[f(t)\vec{u}(t)] =$

4. $\dfrac{d}{dt}[\vec{u}(t) \cdot \vec{v}(t)] =$

5. $\dfrac{d}{dt}[\vec{u}(t) \times \vec{v}(t)] =$

6. $\dfrac{d}{dt}[\vec{u}(f(t))] =$

Example. Show that if $\vec{r}(t)$ has constant length, then $\vec{r}(t)$ and $\vec{r}\,'(t)$ are orthogonal for all t. What does this result mean geometrically?

Definition. What is the definite integral of a continuous vector function $\vec{r}(t)$?

Question. How can we extend the Fundamental Theorem of Calculus to continuous vector functions?

Example. Find $\int_0^{\pi/2} \vec{r}(t)\,dt$ if $\vec{r}(t) = 2\cos t\vec{\imath} + \sin t\vec{\jmath} + 2t\vec{k}$.