Trigonometric Substitution Using Secant

Substitution:

$$x = 2\sec\theta, \quad dx = 2\sec\theta\tan\theta\, d\theta$$
$$\sqrt{x^2-4} = 2\tan\theta$$

Integral Transformations:

Original Integral	Substitution	Transformed Integral
$\int \frac{1}{\sqrt{x^2-4}}\, dx$	$\int \frac{1}{2\tan\theta} \cdot 2\sec\theta\tan\theta\, d\theta$	$\int \sec\theta\, d\theta$
$\int \frac{x}{\sqrt{x^2-4}}\, dx$	$\int \frac{2\sec\theta}{2\tan\theta} \cdot 2\sec\theta\tan\theta\, d\theta$	$\int 2\sec^2\theta\, d\theta$
$\int \frac{x^2}{\sqrt{x^2-4}}\, dx$	$\int \frac{(2\sec\theta)^2}{2\tan\theta} \cdot 2\sec\theta\tan\theta\, d\theta$	$\int 4\sec^3\theta\, d\theta$
$\int \frac{x^3}{\sqrt{x^2-4}}\, dx$	$\int \frac{(2\sec\theta)^3}{2\tan\theta} \cdot 2\sec\theta\tan\theta\, d\theta$	$\int 8\sec^4\theta\, d\theta$
$\int \frac{\sqrt{x^2-4}}{x}\, dx$	$\int \frac{2\tan\theta}{2\sec\theta} \cdot 2\sec\theta\tan\theta\, d\theta$	$\int 2\tan^2\theta\, d\theta$
$\int \frac{\sqrt{x^2-4}}{x^2}\, dx$	$\int \frac{2\tan\theta}{4\sec^2\theta} \cdot 2\sec\theta\tan\theta\, d\theta$	$\int \frac{\tan^2\theta}{\sec\theta}\, d\theta$
$\int \frac{\sqrt{x^2-4}}{x^3}\, dx$	$\int \frac{2\tan\theta}{(2\sec\theta)^3} \cdot 2\sec\theta\tan\theta\, d\theta$	$\int \frac{\tan^2\theta}{2\sec^2\theta}\, d\theta$
$\int \frac{1}{x\sqrt{x^2-4}}\, dx$	$\int \frac{1}{2\sec\theta \cdot 2\tan\theta} \cdot 2\sec\theta\tan\theta\, d\theta$	$\int \frac{1}{2}\, d\theta$
$\int \frac{1}{x^2\sqrt{x^2-4}}\, dx$	$\int \frac{1}{4\sec^2\theta \cdot 2\tan\theta} \cdot 2\sec\theta\tan\theta\, d\theta$	$\int \frac{1}{4\sec\theta}\, d\theta$
$\int \frac{1}{x^3\sqrt{x^2-4}}\, dx$	$\int \frac{1}{8\sec^3\theta \cdot 2\tan\theta} \cdot 2\sec\theta\tan\theta\, d\theta$	$\int \frac{1}{8\sec^2\theta}\, d\theta$

Note: Some integrals, such as $\int \frac{x}{\sqrt{x^2-4}}\, dx$, can also be solved using u-substitution with $u = x^2 - 4$, leading to $du = 2x\, dx$.

Trigonometric Substitution Using Sine

Substitution:

$$x = 2\sin\theta, \quad dx = 2\cos\theta\, d\theta$$
$$\sqrt{4 - x^2} = 2\cos\theta$$

Integral Transformations:

Original Integral	Substitution	Transformed Integral
$\int \frac{1}{\sqrt{4-x^2}}\, dx$	$\int \frac{1}{2\cos\theta} \cdot 2\cos\theta\, d\theta$	$\int d\theta$
$\int \frac{x}{\sqrt{4-x^2}}\, dx$	$\int \frac{2\sin\theta}{2\cos\theta} \cdot 2\cos\theta\, d\theta$	$\int 2\sin\theta\, d\theta$
$\int \frac{x^2}{\sqrt{4-x^2}}\, dx$	$\int \frac{(2\sin\theta)^2}{2\cos\theta} \cdot 2\cos\theta\, d\theta$	$\int 4\sin^2\theta\, d\theta$
$\int \frac{x^3}{\sqrt{4-x^2}}\, dx$	$\int \frac{(2\sin\theta)^3}{2\cos\theta} \cdot 2\cos\theta\, d\theta$	$\int 8\sin^3\theta\, d\theta$
$\int \frac{\sqrt{4-x^2}}{x}\, dx$	$\int \frac{2\cos\theta}{2\sin\theta} \cdot 2\cos\theta\, d\theta$	$\int 2\cos^2\theta \csc\theta\, d\theta$
$\int \frac{\sqrt{4-x^2}}{x^2}\, dx$	$\int \frac{2\cos\theta}{4\sin^2\theta} \cdot 2\cos\theta\, d\theta$	$\int \frac{\cos^2\theta}{\sin^2\theta}\, d\theta$
$\int \frac{\sqrt{4-x^2}}{x^3}\, dx$	$\int \frac{2\cos\theta}{(2\sin\theta)^3} \cdot 2\cos\theta\, d\theta$	$\int \frac{\cos^2\theta}{4\sin^3\theta}\, d\theta$
$\int \frac{1}{x\sqrt{4-x^2}}\, dx$	$\int \frac{1}{2\sin\theta \cdot 2\cos\theta} \cdot 2\cos\theta\, d\theta$	$\int \frac{1}{2\sin\theta}\, d\theta$
$\int \frac{1}{x^2\sqrt{4-x^2}}\, dx$	$\int \frac{1}{4\sin^2\theta \cdot 2\cos\theta} \cdot 2\cos\theta\, d\theta$	$\int \frac{1}{4\sin^2\theta}\, d\theta$
$\int \frac{1}{x^3\sqrt{4-x^2}}\, dx$	$\int \frac{1}{8\sin^3\theta \cdot 2\cos\theta} \cdot 2\cos\theta\, d\theta$	$\int \frac{1}{8\sin^3\theta}\, d\theta$

Note: Some integrals, such as $\int \frac{x}{\sqrt{4-x^2}}\, dx$, can also be solved using u-substitution with $u = 4 - x^2$, leading to $du = -2x\, dx$. Additionally, $\int \frac{1}{\sqrt{4-x^2}}\, dx$ can be solved using $\arcsin(\theta)$.

Trigonometric Substitution Using Tangent

Substitution:

$$x = 2\tan\theta, \quad dx = 2\sec^2\theta\, d\theta$$
$$\sqrt{x^2+4} = 2\sec\theta$$

Integral Transformations:

Original Integral	Substitution	Transformed Integral
$\int \frac{1}{\sqrt{x^2+4}}\, dx$	$\int \frac{1}{2\sec\theta} \cdot 2\sec^2\theta\, d\theta$	$\int \sec\theta\, d\theta$
$\int \frac{x}{\sqrt{x^2+4}}\, dx$	$\int \frac{2\tan\theta}{2\sec\theta} \cdot 2\sec^2\theta\, d\theta$	$\int 2\sec\theta\tan\theta\, d\theta$
$\int \frac{x^2}{\sqrt{x^2+4}}\, dx$	$\int \frac{(2\tan\theta)^2}{2\sec\theta} \cdot 2\sec^2\theta\, d\theta$	$\int 4\sec\theta\tan^2\theta\, d\theta$
$\int \frac{x^3}{\sqrt{x^2+4}}\, dx$	$\int \frac{(2\tan\theta)^3}{2\sec\theta} \cdot 2\sec^2\theta\, d\theta$	$\int 8\sec\theta\tan^3\theta\, d\theta$
$\int \frac{\sqrt{x^2+4}}{x}\, dx$	$\int \frac{2\sec\theta}{2\tan\theta} \cdot 2\sec^2\theta\, d\theta$	$\int \frac{2\sec^3\theta}{\tan\theta}\, d\theta$
$\int \frac{\sqrt{x^2+4}}{x^2}\, dx$	$\int \frac{2\sec\theta}{4\tan^2\theta} \cdot 2\sec^2\theta\, d\theta$	$\int \frac{\sec^3\theta}{\tan^2\theta}\, d\theta$
$\int \frac{\sqrt{x^2+4}}{x^3}\, dx$	$\int \frac{2\sec\theta}{8\tan^3\theta} \cdot 2\sec^2\theta\, d\theta$	$\int \frac{\sec^3\theta}{2\tan^3\theta}\, d\theta$
$\int \frac{1}{x\sqrt{x^2+4}}\, dx$	$\int \frac{1}{2\tan\theta \cdot 2\sec\theta} \cdot 2\sec^2\theta\, d\theta$	$\int \frac{\sec\theta}{2\tan\theta}\, d\theta$
$\int \frac{1}{x^2\sqrt{x^2+4}}\, dx$	$\int \frac{1}{4\tan^2\theta \cdot 2\sec\theta} \cdot 2\sec^2\theta\, d\theta$	$\int \frac{\sec\theta}{4\tan^2\theta}\, d\theta$
$\int \frac{1}{x^3\sqrt{x^2+4}}\, dx$	$\int \frac{1}{8\tan^3\theta \cdot 2\sec\theta} \cdot 2\sec^2\theta\, d\theta$	$\int \frac{\sec\theta}{8\tan^3\theta}\, d\theta$

Note: Some integrals, such as $\int \frac{x}{\sqrt{x^2+4}}\, dx$, can also be solved using u-substitution with $u = x^2 + 4$, leading to $du = 2x\, dx$.