How to Determine Whether a Series Converges or Diverges

1. Test for Divergence

If $\lim_{n\to\infty} a_n \neq 0$ or the limit does not exist, then $\sum a_n$ diverges. If $\lim_{n\to\infty} a_n = 0$, this test is inconclusive.

2. Known Series Types

Telescoping Write out several partial sums and look for cancellation. The series converges when the partial sums approach a finite limit.

Geometric $\sum ar^{n-1}$. Converges if $|r| < 1$; diverges if $|r| \geq 1$.

***p*-Series** $\sum \frac{1}{n^p}$. Converges if $p > 1$; diverges if $p \leq 1$.

3. Positive-Term Series $a_n \geq 0$

DCT If $0 \leq a_n \leq b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges. If $0 \leq b_n \leq a_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges. Use when you can compare a_n by inequality with a known p-series or geometric series.

LCT If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, where $0 < c < \infty$, then $\sum a_n$ and $\sum b_n$ have the same behavior. Use when direct comparison is difficult.

Integral Test Use when $a_n = f(n)$, where f is positive, continuous, and decreasing. Then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ have the same behavior.

Ratio Test Best for factorials, exponentials, and powers. Let $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$. If $L < 1$, the series converges absolutely; if $L > 1$, it diverges; if $L = 1$, the test is inconclusive.

4. Mixed-Sign Series

Absolute convergence If $\sum |a_n|$ converges, then $\sum a_n$ also converges. In this case, $\sum a_n$ is called absolutely convergent. Use the tests on the left to study $\sum |a_n|$.

Alternating Series Test If $\sum a_n$ has the form $\sum (-1)^n b_n$ or $\sum (-1)^{n+1} b_n$, where $b_n > 0$, and if $b_n \to 0$ and b_n is eventually decreasing, then $\sum a_n$ converges.

Series Classification

Absolutely convergent	$\sum |a_n|$ converges.
Conditionally convergent	$\sum a_n$ converges, but $\sum |a_n|$ diverges.
Divergent	$\sum a_n$ diverges.