Math 2300
Spring 2026
Quiz 9

Name: Champ

1. (1 point) Determine whether the following geometric series converges. If it does, find its sum.

$$\sum_{n=2}^{\infty} \frac{18}{3^n} = \sum_{n=2}^{\infty} 18 \cdot \left(\tfrac{1}{3}\right)^n$$

(a) 3
(b) 6
(c) 9
(d) 27
(e) Does not converge

Since $|r| = \frac{1}{3} < 1$, this converges to

$$\frac{\text{first term}}{1-r} = \frac{2}{1-1/3} = \frac{2}{2/3} = 2 \cdot \frac{3}{2} = 3$$

2. (1 point) Which of the following series is a convergent p-series?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{-1/2}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/4}}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{5/4}}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{0.99}}$

p-series converges when $p > 1$

3. (1 point) Determine which statement best justifies the behavior of the series

$$\sum_{n=3}^{\infty} \frac{7}{4^n - n^2}.$$

This series:

(a) Converges by direct comparison with $b_n = \dfrac{7}{4^n}$. ← inequality wrong way
(b) Converges by limit comparison with $b_n = \dfrac{1}{4^n}$.
(c) Converges by limit comparison with $b_n = \dfrac{1}{n^2}$. ← inconclusive (get 0 for limit)
(d) Converges because it is a geometric series with common ratio $r = \dfrac{1}{4}$. ← not geometric
(e) Diverges by the divergence test. ← the terms DO go to 0

4. (4 points) Determine whether the following series converges or diverges. Fully justify your answer.

$$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$$

Introduction: The function $f(x) = \frac{1}{x\ln(x)}$ is positive and continuous for $x > 1$. To show $f(x)$ is decreasing, we compute the derivative:

$$f'(x) = -(x\ln(x))^{-2} \cdot \left(x \cdot \frac{1}{x} + \ln(x)\right) = -\frac{1+\ln(x)}{(x\ln(x))^2}$$

$f'(x) < 0$ when $x > 2$. The Integral Test applies.

Apply the Test:

$$\int_2^{\infty} \frac{1}{x\ln(x)}\,dx = \lim_{t\to\infty} \int_2^{t} \frac{1}{x\ln(x)}\,dx = \lim_{t\to\infty} \int_{\ln(2)}^{\ln(t)} \frac{1}{u}\,du \qquad \left(\begin{array}{l} u = \ln x \\ du = \frac{1}{x}\,dx \end{array}\right)$$

$$= \lim_{t\to\infty} \Big[\ln(u)\Big]_{\ln(2)}^{\ln(t)} = \lim_{t\to\infty} \Big(\ln(\ln(t)) - \ln(\ln(2))\Big) = \infty$$

Conclusion: Since $\int_2^{\infty} \frac{1}{x\ln(x)}\,dx$ diverges, the series $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ diverges by the Integral Test.

5. (3 points) Do you have any questions or comments about the course so far?

I love MATH 2300!