Math 2300 **Name:** Champ
Spring 2026
Quiz 8

1. (2 points) Determine whether the following sequence converges, and if it does, find its limit.

$$a_n = 1 - \frac{3\sin(n)}{2^n}.$$

(a) The sequence converges to 0.
(b) The sequence converges to 1.
(c) The sequence converges to $\frac{3}{2}$.
(d) The sequence converges to 3.
(e) The sequence diverges.

Note: Since $\frac{-1}{2^n} \le \frac{\sin(n)}{2^n} \le \frac{1}{2^n}$, $\frac{\sin(n)}{2^n} \to 0$ by the squeeze theorem.

2. (2 points) Suppose a series $\sum_{k=1}^{\infty} a_k$ has nth partial sum

$$S_n = \sum_{k=1}^{n} a_k = \frac{5n^4}{1+2n^4} \qquad (n \geq 1).$$

Which of the following statements *must* be true?

(a) The series converges, but its sum cannot be determined.
(b) The series converges, and $\sum_{k=1}^{\infty} a_k = 0$.
(c) The series converges, and $\sum_{k=1}^{\infty} a_k = \frac{5}{2}$.
(d) The series diverges by the Divergence Test because $\lim_{n\to\infty} a_n \neq 0$.
(e) There is not enough information to determine whether $\sum_{k=1}^{\infty} a_k$ converges.

3. (2 points) Consider the series $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$. Which of the following statements is true?

(a) The series converges to 0.
(b) The series converges to 1.
(c) The series converges to a value other than 0 or 1.
(d) The series diverges because the sequence of terms approaches 0.
(e) The series diverges because the sequence of terms approaches 1.

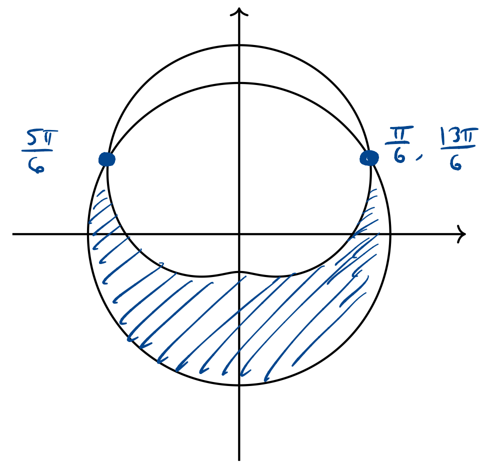
4. (4 points) Find the area of the region that lies inside the graph of $r = 4$ and outside the graph of $r = 3 + 2\sin\theta$.

Intersection Points:

$4 = 3 + 2\sin\theta$

$1 = 2\sin\theta$

$\frac{1}{2} = \sin\theta \implies \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$



$$A = \frac{1}{2}\int_{5\pi/6}^{13\pi/6} 4^2 - (3+2\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_{5\pi/6}^{13\pi/6} 16 - [9 + 12\sin\theta + 4\sin^2\theta] \, d\theta$$

$$= \frac{1}{2}\int_{5\pi/6}^{13\pi/6} 7 - 12\sin\theta - 4\sin^2\theta \, d\theta$$

$$= \frac{1}{2}\int_{5\pi/6}^{13\pi/6} 7 - 12\sin\theta - 4\cdot\frac{1}{2}(1-\cos 2\theta) \, d\theta$$

$$= \frac{1}{2}\int_{5\pi/6}^{13\pi/6} 5 - 12\sin\theta + 2\cos 2\theta \, d\theta$$

$$= \frac{1}{2}\Big[5\theta + 12\cos\theta + \sin 2\theta\Big]_{5\pi/6}^{13\pi/6}$$

$$= \frac{1}{2}\left[5\cdot\frac{13\pi}{6} + 12\cos\left(\frac{13\pi}{6}\right) + \sin\left(\frac{13\pi}{3}\right)\right] - \frac{1}{2}\left[5\cdot\frac{5\pi}{6} + 12\cos\left(\frac{5\pi}{6}\right) + \sin\left(\frac{5\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[\frac{65\pi}{6} + 12\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right] - \frac{1}{2}\left[\frac{25\pi}{6} - 12\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right]$$

$$= \frac{65\pi}{12} + \frac{13\sqrt{3}}{4} - \frac{25\pi}{12} + \frac{13\sqrt{3}}{4} = \boxed{\frac{10\pi}{3} + \frac{13\sqrt{3}}{2}}$$