Math 2300
Spring 2026
Quiz 7

Name: Champ

1. (2 points) Consider the parametric curve

$$x = \frac{1}{2}t^2, \qquad y = \sqrt{t}, \qquad 0 \le t \le 6.$$

What is the equation of the line tangent to the curve at the point $(8, 2)$? $\nearrow t = 4$

(a) $y = \frac{1}{4}x$

(b) $y = \frac{1}{8}x + 1$

(c) $y = -\frac{1}{16}x + \frac{5}{2}$

(d) $y = 16x + 2$

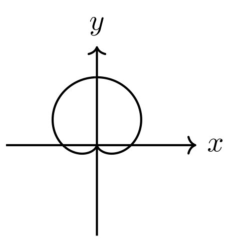
(e) $y = \frac{1}{16}x + \frac{3}{2}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{1}{2}t^{-1/2}}{t} = \frac{1}{2t^{3/2}}$$

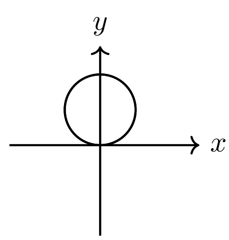
When $t = 4$, $\frac{dy}{dx} = \frac{1}{2\cdot(\sqrt{4})^3} = \frac{1}{16}$

$$y - 2 = \frac{1}{16}(x - 8) \Rightarrow y = \frac{1}{16}x + \frac{3}{2}$$

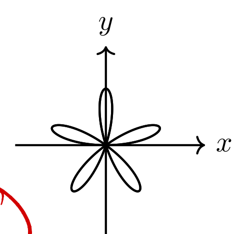
2. (2 points) Which of the following graphs represents the polar function $r = \sin(5\theta)$?



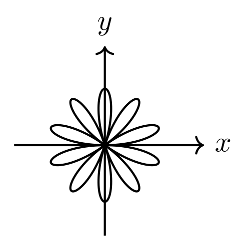
(a)



(b)



(c)



(d)

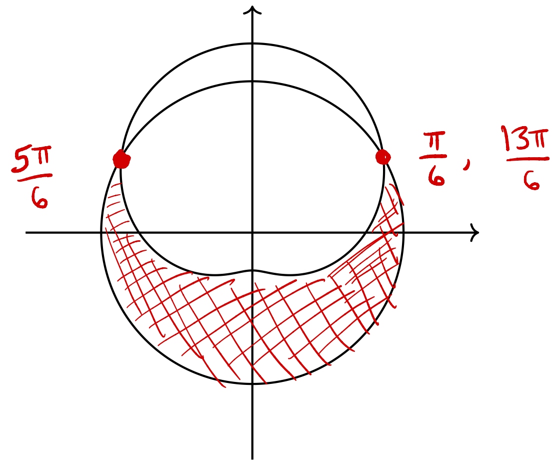
3. (2 points) Set up an integral for the arc length of the parametric curve given by

$$\begin{cases} x(t) = \sin(t), \\ y(t) = t^3, \end{cases} \qquad 0 \le t \le 2\pi.$$

You **do not** have to compute the integral.

$$L = \boxed{\int_0^{2\pi} \sqrt{(\cos t)^2 + (3t^2)^2}\, dt}$$

4. (4 points) **Set up, but do not evaluate**, an integral that computes the area of the region that lies inside the graph of $r = 4$ and outside the graph of $r = 3 + 2\sin\theta$.



Intersection Points:

$4 = 3 + 2\sin\theta$

$1 = 2\sin\theta$

$\frac{1}{2} = \sin\theta \Rightarrow \theta = \pi/6,\ 5\pi/6,\ 13\pi/6$

⚠ Warning. Integrating from $\pi/6$ to $5\pi/6$ would give you the wrong region! Need angles where rays between the two sweep out the bottom of the circle.

Integral:

$$\frac{1}{2}\int_{5\pi/6}^{13\pi/6} 4^2\, d\theta - \frac{1}{2}\int_{5\pi/6}^{13\pi/6} (3+2\sin\theta)^2\, d\theta$$