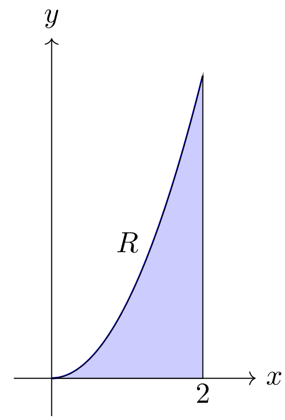
Math 2300 **Name:** Champ
Spring 2026
Quiz 6

Must know formulas for $\bar{x}$ and $\bar{y}$

1. (2 points) Consider the region R shown on the right, which is bounded by the curve $y = x^2$, the x-axis, and the line $x = 2$. The area of the region is

$$A = \int_0^2 x^2\,dx = \frac{8}{3}.$$

Which of the following is the correct value of $\bar{x}$, the x-coordinate of the center of mass?



(a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) $\frac{8}{5}$ (d) $\frac{5}{3}$ (e) $\frac{7}{4}$

$$\bar{x} = \frac{1}{A}\int_a^b x \cdot f(x)\,dx = \frac{3}{8}\int_0^2 x^3\,dx = \frac{3}{8}\left[\frac{1}{4}x^4\right]_0^2 = \frac{3}{8}\cdot 4 = \frac{3}{2}$$

2. (2 points) Which of the following sets of parametric equations describes a circle of radius 4, centered at $(1, -3)$, that is traced exactly once in a **counterclockwise** direction for $0 \le t \le 2\pi$?

(a) $x = 1 + 4\cos(-t), \quad y = -3 + 4\sin(-t)$

(b) $x = 1 + 4\cos(\pi - t), \quad y = -3 + 4\sin(\pi - t)$

(c) $x = 1 + 4\sin t, \quad y = -3 + 4\cos t$

(d) $x = 1 + 4\cos t, \quad y = -3 + 4\sin t$

(e) $x = 1 + 4\cos(2t), \quad y = -3 + 4\sin(2t)$

3. (2 points) Given the parametric equations

$$x(t) = e^t - 2, \quad y(t) = t^2 + t - 3,$$

find the equation of the tangent line to the curve when $t = 0$.

(a) $y = x - 1$

(b) $y = x + 2$

(c) $y = x - 2$

(d) $y = 2x - 3$

(e) $y = -2x - 3$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t+1}{e^t}$$

when $t = 0$, $\frac{dy}{dx} = 1$ and $x = -1$, $y = -3$

$y + 3 = x + 1 \Rightarrow y = x - 2$

4. (4 points) Consider the parametric equations

$$x(t) = t^2, \quad y(t) = 2t^2, \quad t \geq 0.$$

Find the arc length of the curve from $(0,0)$ to $(4,8)$.

$t=0 \qquad t=2$

Method 1:

$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^2 \sqrt{(2t)^2 + (4t)^2}\, dt$$

$$= \int_0^2 \sqrt{4t^2 + 16t^2}\, dt$$

$$= \int_0^2 \sqrt{20t^2}\, dt$$

$$= \int_0^2 \sqrt{20}\, t\, dt = \sqrt{20}\left[\tfrac{1}{2}t^2\right]_0^2 = \sqrt{20} \cdot 2$$

Method 2:

$$dy/dx = \frac{dy/dt}{dx/dt} = \frac{4t}{2t} = 2$$

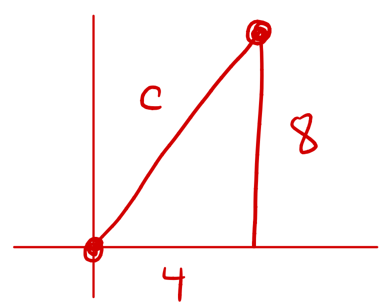
$$L = \int_a^b \sqrt{1 + (dy/dx)^2}\, dx$$

$$= \int_0^4 \sqrt{1+4}\, dx$$

$$= 4\sqrt{5}$$

Method 3:

Note: $y = 2x$



$$c = \sqrt{4^2 + 8^2} = \sqrt{80}$$