Math 2300
Spring 2026
Quiz 5

Name: _____

1. (1 point) Which of the following integrals represents the volume of the solid of revolution formed by rotating the region bounded by the x-axis and $f(x) = x - x^2$ around the y-axis using the cylindrical shells method?

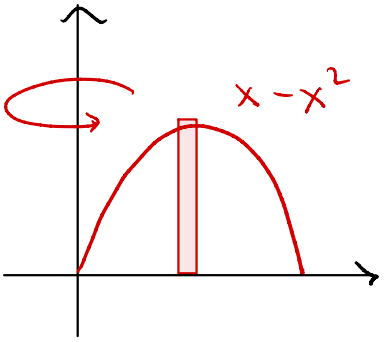
   (a) $\int_0^1 \pi x^2(x - x^2)\,dx$

   (b) $\int_0^1 2\pi x(x - x^2)\,dx$

   (c) $\int_0^1 \pi(x - x^2)^2\,dx$

   (d) $\int_0^1 2\pi(x - x^2)\,dx$

   (e) $\int_0^1 2\pi x^2(x - x^2)\,dx$



$\int_0^1 2\pi x \cdot (x - x^2) \cdot dx$
circumference height thickness

2. (1 point) A 80 foot rope weighing 10 lb per foot hangs freely over the side of a ship. The crew pulls the rope straight up onto the deck until the entire rope has been wound in. Choose the integral that computes the work done in pulling in the rope.

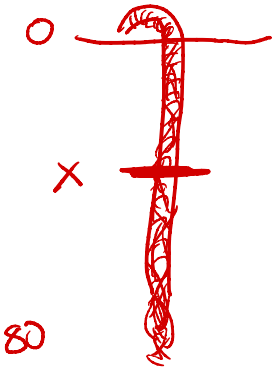
   (a) $\int_0^{80} 10(80 - x)^2\,dx$

   (b) $\int_0^{80} 800\,dx$

   (c) $\int_0^{80} 10x\,dx$

   (d) $\int_0^{80} x\,dx$

   (e) $\int_0^{80} 10x^2\,dx$



$\int_0^{80} x \cdot 10\,dx$
distance force (weight)

3. (1 point) A system of three point masses lies in the plane. A mass of 2 kg is located at $(0, 0)$, a mass of 3 kg is located at $(4, 0)$, and a mass of 5 kg is located at $(0, 2)$. What is the center of mass of the system?

   (a) (2, 1)

   (b) $\left(\frac{6}{5}, 1\right)$

   (c) $\left(\frac{4}{3}, \frac{2}{5}\right)$

   (d) $\left(\frac{2}{3}, \frac{5}{2}\right)$

   (e) $\left(\frac{3}{5}, \frac{2}{7}\right)$

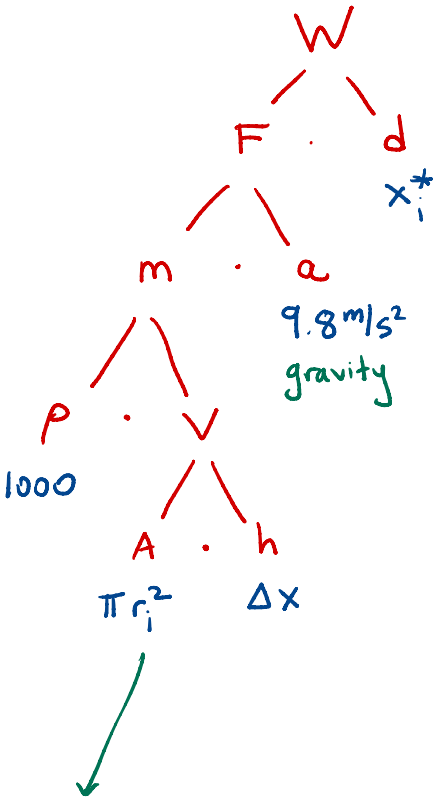
$M_y = 2(0) + 3(4) + 5(0) = 12$

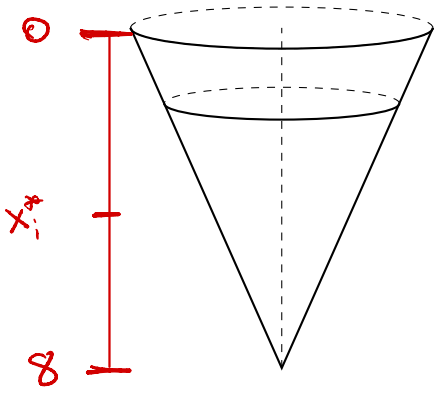
$M_x = 2(0) + 3(0) + 5(2) = 10$

$m = 2 + 3 + 5 = 10$

$\bar{x} = 12/10 = 6/5, \quad \bar{y} = 10/10 = 1$

4. (4 points) A tank has the shape of an inverted circular cone with height 8 m and base radius 4 m. It is filled with water to a height of 6 m (measured upward from the tip). How much work is required to pump all of the water to the top rim of the tank? **Set up, but do not evaluate, an integral for the work required.** Assume the density of water is 1000 kg/m^3 and take $g = 9.8$ m/s^2.





$$W_i = F_i \cdot d_i$$

$$= 1000\pi \left(\frac{1}{2}(8 - x_i^*)\right)^2 \cdot 9.8 \cdot x_i^* \, \Delta x$$

$$= 2450\pi (8 - x_i^*)^2 \cdot x_i^* \, \Delta x$$

$$W_{\text{Total}} = \lim_{n\to\infty} \sum_{i=1}^{n} 2450\pi (8 - x_i^*)^2 \cdot x_i^* \, \Delta x$$

$$= \int_2^8 2450\pi (8 - x)^2 x \, dx$$

By similar triangles,

$$\frac{r_i}{8 - x_i^*} = \frac{4}{8} \Rightarrow r_i = \frac{1}{2}(8 - x_i^*)$$

5. (3 points) Do you have any questions or comments about the course so far?

MATH 2300 is awesome!