Math 2300
Spring 2026
Quiz 4

Name: Champ

1. (2 points) Evaluate the definite integral.

$$\int_{-2}^{-1} x\sqrt{x+2}\,dx$$

Let $u = x+2$, $du = dx$

$\int_0^1 (u-2)\sqrt{u}\,du = \int_0^1 u^{3/2} - 2u^{1/2}\,du$

$\left[\frac{2}{5}u^{5/2} - 2\cdot\frac{2}{3}u^{3/2}\right]_0^1 = \frac{2}{5} - \frac{4}{3} = \frac{6}{15} - \frac{20}{15}$

(a) $-\dfrac{14}{15}$ (b) $\dfrac{14}{15}$ (c) $-\dfrac{2}{5}$ (d) $-\dfrac{4}{3}$ (e) $\dfrac{32}{15}$

2. (2 points) Determine whether the improper integral converges or diverges. If it converges, find its value.

$$\int_{-2}^{2} \frac{1}{x^2}\,dx = \int_{-2}^{0} \frac{1}{x^2}\,dx + \int_{0}^{2} \frac{1}{x^2}\,dx$$

(a) It converges to 0.
(b) It converges to $\frac{1}{2}$.
(c) It converges to 1.
(d) It diverges.
(e) There is not enough information to decide.

$\Rightarrow \int_{-2}^{0} \frac{1}{x^2}\,dx = \lim_{t\to 0^-} \int_{-2}^{t} x^{-2}\,dx$

$= \lim_{t\to 0^-} \left[-x^{-1}\right]_{-2}^{t} = \lim_{t\to 0^-} \left[-\frac{1}{t} + \frac{1}{2}\right]$

diverges

3. (2 points) Let R be the region between $y = f(x)$ and $y = g(x)$ on $0 \le x \le 2$, where

$$f(x) = 2x+1, \qquad g(x) = x^2.$$

A solid has base R, and cross-sections perpendicular to the x-axis are squares. Which definite integral represents the volume of the solid?

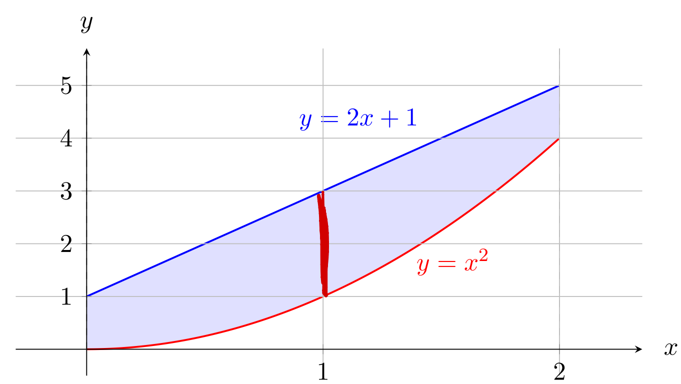
(a) $\displaystyle\int_0^2 \left[(2x+1) - x^2\right]^2 dx$

(b) $\displaystyle\int_0^2 \left[(2x+1) - x^2\right] dx$

(c) $\displaystyle\int_0^2 \left[(2x+1)^2 - (x^2)^2\right] dx$

(d) $\displaystyle\int_0^2 \pi\left[(2x+1) - x^2\right]^2 dx$

(e) $\displaystyle\int_2^0 \left[(2x+1) - x^2\right]^2 dx$



$A(x) = [s(x)]^2 = \left[(2x+1) - x^2\right]^2$

4. (4 points) Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{\cos x}{y}, \qquad y(0) = 3.$$

$y\,dy = \cos x\,dx$

$\int y\,dy = \int \cos x\,dx$

$\frac{1}{2}y^2 = \sin x + C$

$y^2 = 2\sin x + 2C$

$y^2 = 2\sin x + K$

$9 = 2\sin(0) + K \Rightarrow K = 9$

$y^2 = 2\sin x + 9$

$y = \pm\sqrt{2\sin x + 9}$

$\boxed{y = \sqrt{2\sin x + 9}}$