Math 2300 **Name:** Champ
Spring 2026
Quiz 3

1. (2 points) Choose the correct partial fractions expansion for the function

$$f(x) = \frac{7x^3 - 5x^2 + 2x + 9}{(x-2)^2(x+1)(2x-3)(x^2+4)}.$$

(a) $\dfrac{A}{x-2} + \dfrac{B}{x-2} + \dfrac{C}{x+1} + \dfrac{D}{2x-3} + \dfrac{Ex+F}{x^2+4}$

(b) $\dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{x+1} + \dfrac{D}{2x-3} + \dfrac{E}{x^2+4}$

(c) $\dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{x+1} + \dfrac{D}{2x-3} + \dfrac{Ex+F}{x^2+4}$

(d) $\dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{Cx+D}{x+1} + \dfrac{E}{2x-3} + \dfrac{F}{x^2+4}$

(e) $\dfrac{Ax+B}{(x-2)^2} + \dfrac{C}{x+1} + \dfrac{D}{2x-3} + \dfrac{Ex+F}{x^2+4}$

2. (2 points) Which of the following options is equal to the given integral?

$$\int \left(\frac{5}{2x+7} + \frac{8}{x^2+4} \right) dx$$

(a) $\dfrac{-10}{(2x+7)^2} - \dfrac{16x}{(x^2+4)^2} + C$

(b) $\dfrac{5}{2} \ln|2x+7| + 4\ln(x^2+4) + C$

(c) $5\ln|2x+7| + 8\arcsin\left(\dfrac{x}{2}\right) + C$

(d) $\dfrac{5}{2}\ln|2x+7| + 4\arctan\left(\dfrac{x}{2}\right) + C$

(e) $5(2x+7) + \dfrac{8}{x}\arctan(x) + C$

3. (2 points) Suppose the functions f and g below are continuous on $(0, \infty)$ and $f(x) > g(x) > 0$ for all $x > 1$. If you know that $\displaystyle\int_1^\infty f(x)\,dx$ converges, what can you conclude about g?

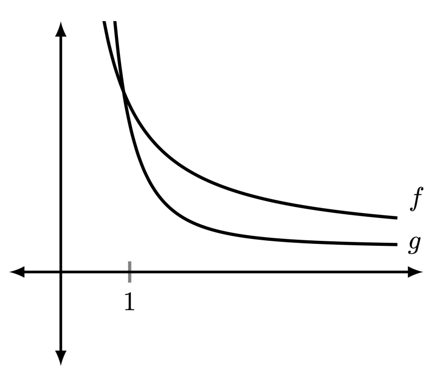
(a) $\int_1^\infty g(x)\,dx = 0$

(b) $\int_1^\infty g(x)\,dx$ converges

(c) $\int_1^\infty g(x)\,dx = \int_1^\infty f(x)\,dx$

(d) $\int_1^\infty g(x)\,dx$ diverges

(e) You cannot conclude anything



4. (4 points) Evaluate the integral.

$$\int_0^1 \frac{2x^3}{(x^2+1)^3}\,dx$$

U-substitution

$u = x^2 + 1 \quad \Rightarrow \quad x^2 = u - 1$

$du = 2x \cdot dx$

$$\int_0^1 \frac{x^2}{(x^2+1)^3} \cdot 2x\,dx$$

$$\int_1^2 \frac{u-1}{u^3}\,du$$

$$\int_1^2 u^{-2} - u^{-3}\,du$$

$$\left[\frac{u^{-1}}{-1} - \frac{u^{-2}}{-2} \right]_1^2$$

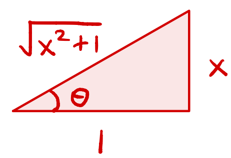
$$\left[-\frac{1}{u} + \frac{1}{2u^2} \right]_1^2$$

$$\left(-\frac{1}{2} + \frac{1}{8}\right) - \left(-1 + \frac{1}{2}\right)$$

$$-\frac{1}{2} + \frac{1}{8} + 1 - \frac{1}{2}$$

$$\boxed{\frac{1}{8}}$$

Trig Substitution



$x = \tan\theta \qquad dx = \sec^2\theta \qquad \sqrt{x^2+1} = \sec\theta$

$$\int_0^{\pi/4} \frac{2\tan^3\theta}{\sec^6\theta} \cdot \sec^2\theta\,d\theta$$

$$2\int_0^{\pi/4} \frac{\tan^3\theta}{\sec^4\theta}\,d\theta$$

$$2\int_0^{\pi/4} \frac{\sin^3\theta}{\cos^3\theta} \cdot \cos^4\theta\,d\theta$$

$$2\int_0^{\pi/4} \sin^3\theta\cos\theta\,d\theta$$

$u = \sin\theta, \quad du = \cos\theta\,d\theta$

$$2\int_0^{\sqrt{2}/2} u^3\,du$$

$$2\left[\frac{1}{4}u^4\right]_0^{\sqrt{2}/2}$$

$$2\left[\frac{1}{4}\cdot\left(\frac{\sqrt{2}}{2}\right)^4\right]$$

$$2 \cdot \frac{1}{4} \cdot \frac{4}{16}$$

$$\boxed{\frac{1}{8}}$$

4. (4 points) Evaluate the integral.

$$\int_0^1 \frac{2x^3}{(x^2+1)^3}\,dx$$

Partial Fractions:

$$\frac{2x^3}{(x^2+1)^3} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2} + \frac{Ex+F}{(x^2+1)^3}$$

$$2x^3 = (Ax+B)(x^2+1)^2 + (Cx+D)(x^2+1) + Ex+F$$

$$= (Ax+B)(x^4+2x^2+1) + (Cx^3+Cx+Dx^2+D) + Ex+F$$

$$= \cancel{Ax^5} + \cancel{2Ax^3} + \cancel{Ax} + \cancel{Bx^4} + \cancel{2Bx^2} + \cancel{B} + \cancel{Cx^3} + \cancel{Cx} + \cancel{Dx^2} + \cancel{D} + \cancel{Ex} + \cancel{F}$$

$$= Ax^5 + Bx^4 + (2A+C)x^3 + (2B+D)x^2 + (A+E)x + B+D+F$$

$A = 0$
$B = 0$
$2A + C = 2$
$2B + D = 0$
$A + E + C = 0$
$B + D + F = 0$

$\Rightarrow$

$A = 0$
$B = 0$
$C = 2$
$D = 0$
$E = -2$
$F = 0$

$$\int_0^1 \frac{2x}{(x^2+1)^2} - \frac{2x}{(x^2+1)^3}\,dx$$

Let $u = x^2+1$, $du = 2x$

$$\int_1^2 \frac{1}{u^2}\,du - \int_1^2 \frac{1}{u^3}\,du$$

$$\left[\frac{u^{-1}}{-1}\right]_1^2 - \left[\frac{u^{-2}}{-2}\right]_1^2$$

$$\left(-\frac{1}{2} + \frac{1}{1}\right) - \left(\frac{1}{-2\cdot 4} + \frac{1}{2\cdot 1}\right)$$

$$-\frac{1}{2} + 1 + \frac{1}{8} - \frac{1}{2}$$

$$\boxed{\frac{1}{8}}$$