Quiz 3 Outline

Format. This quiz has **3 multiple-choice** questions and **1 free-response** question.

1. (2 points) Choose the correct partial fractions expansion.

Example: Choose the correct partial fractions expansion for the function

$$f(x) = \frac{10 - 3x^2}{(x-3)^2(x^2+1)(2x+1)}.$$

A. $\frac{A}{x-3} + \frac{Bx+C}{x^2+1} + \frac{D}{2x+1}$

B. $\frac{A}{x-3} + \frac{B}{x-3} + \frac{Cx}{x^2+1} + \frac{D}{2x+1}$

C. $\frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{x^2+1} + \frac{D}{2x+1}$

D. $\frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{Cx+D}{x^2+1} + \frac{E}{2x+1}$

E. $\frac{A}{x-3} + \frac{B}{x-3} + \frac{Cx+D}{x^2+1} + \frac{E}{2x+1}$

Spring 2023 Exam 1 #3

2. (2 points) Integrate term-by-term after partial fraction decomposition.

Example: Which of the following options is equal to the given integral?

$$\int \left(\frac{3}{x-2} + \frac{5}{(x-2)^2} + \frac{2x}{x^2+4} - \frac{1}{x^2+4} \right) dx$$

A. $3\ln|x-2| - \frac{5}{x-2} + \ln(x^2+4) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

B. $3\ln|x-2| + \frac{5}{x-2} + \ln(x^2+4) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

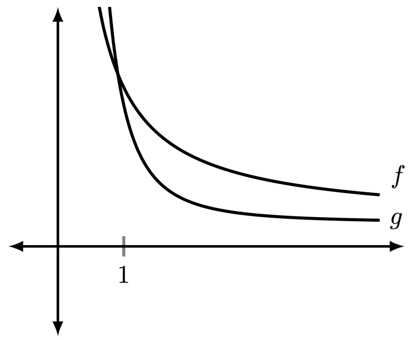
C. $3\ln|x-2| - \frac{5}{x-2} + \ln(x^2+4) - \arctan(x) + C$

D. $3\ln|x-2| - \frac{5}{x-2} + \ln(x^2+4) - \frac{1}{2}\ln(x^2+4) + C$

E. $3\ln|x-2| - \frac{5}{(x-2)^2} + \ln(x^2+4) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

3. (2 points) Apply the Comparison Theorem for improper integrals (Thursday project).

Example: Suppose the functions f and g below are continuous on $(0, \infty)$ and $f(x) > g(x) > 0$ for all $x > 1$. If you know that $\int_1^\infty g(x)\,dx$ diverges, what can you conclude about f?



A. $\int_1^\infty f(x)\,dx = 0$

B. $\int_1^\infty f(x)\,dx$ converges

C. $\int_1^\infty f(x)\,dx = \int_1^\infty g(x)\,dx$

D. $\int_1^\infty f(x)\,dx$ diverges

E. You cannot conclude anything

Spring 2023 Exam 1 #5

4. (4 points) The free response problem will be an integral where you have to decide between u-substitution, trig substitution, or partial fractions.

Examples:

- $\int_0^1 2x^3\left(1-x^2\right)^{1/3}\,dx$ *Fall 2022 Exam 1 #9*
- $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$ *Fall 2022 Exam 1 #10*
- $\int \frac{x-x^2}{(1+x)(x^2+1)}\,dx$ *Fall 2022 Exam 1 #7*

1. Choose the correct partial fractions expansion for the function

$$f(x) = \frac{10 - 3x^2}{(x-3)^2(x^2+1)(2x+1)}.$$

A. $\dfrac{A}{x-3} + \dfrac{Bx+C}{x^2+1} + \dfrac{D}{2x+1}$

B. $\dfrac{A}{x-3} + \dfrac{B}{x-3} + \dfrac{Cx}{x^2+1} + \dfrac{D}{2x+1}$

C. $\dfrac{A}{x-3} + \dfrac{B}{(x-3)^2} + \dfrac{C}{x^2+1} + \dfrac{D}{2x+1}$

D. $\dfrac{A}{x-3} + \dfrac{B}{(x-3)^2} + \dfrac{Cx+D}{x^2+1} + \dfrac{E}{2x+1}$

E. $\dfrac{A}{x-3} + \dfrac{B}{x-3} + \dfrac{Cx+D}{x^2+1} + \dfrac{E}{2x+1}$

Solution: Factor the denominator:

$$(x-3)^2(x^2+1)(2x+1).$$

This has:

- a repeated linear factor $(x-3)^2$, which contributes
$$\frac{A}{x-3} + \frac{B}{(x-3)^2},$$
- an irreducible quadratic factor (x^2+1), which contributes
$$\frac{Cx+D}{x^2+1},$$
- a linear factor $(2x+1)$, which contributes
$$\frac{E}{2x+1}.$$

Therefore the correct form is

$$\frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{Cx+D}{x^2+1} + \frac{E}{2x+1},$$

which is choice **D**.

2. Which of the following options is equal to the given integral?

$$\int \left(\frac{3}{x-2} + \frac{5}{(x-2)^2} + \frac{2x}{x^2+4} - \frac{1}{x^2+4} \right) dx$$

A. $3\ln|x-2| - \dfrac{5}{x-2} + \ln(x^2+4) - \dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right) + C$

B. $3\ln|x-2| + \dfrac{5}{x-2} + \ln(x^2+4) - \dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right) + C$

C. $3\ln|x-2| - \dfrac{5}{x-2} + \ln(x^2+4) - \arctan(x) + C$

D. $3\ln|x-2| - \dfrac{5}{x-2} + \ln(x^2+4) - \dfrac{1}{2}\ln(x^2+4) + C$

E. $3\ln|x-2| - \dfrac{5}{(x-2)^2} + \ln(x^2+4) - \dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right) + C$

Solution: Integrate term-by-term:

$$\int \frac{3}{x-2}\,dx = 3\ln|x-2|, \qquad \int \frac{5}{(x-2)^2}\,dx = 5\int (x-2)^{-2}\,dx = -\frac{5}{x-2}.$$

Also,

$$\int \frac{2x}{x^2+4}\,dx = \ln(x^2+4),$$

and

$$\int \frac{1}{x^2+4}\,dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right).$$

Since the integrand has $-\dfrac{1}{x^2+4}$, this contributes

$$-\frac{1}{2}\arctan\left(\frac{x}{2}\right).$$

Putting everything together,

$$3\ln|x-2| - \frac{5}{x-2} + \ln(x^2+4) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C,$$

which is choice **A**.

3. Suppose the functions f and g below are continuous on $(0, \infty)$ and $f(x) > g(x) > 0$ for all $x > 1$. If you know that $\int_1^\infty g(x)\,dx$ diverges, what can you conclude about f?

A. $\int_1^\infty f(x)\,dx = 0$

B. $\int_1^\infty f(x)\,dx$ converges

C. $\int_1^\infty f(x)\,dx = \int_1^\infty g(x)\,dx$

D. $\int_1^\infty f(x)\,dx$ diverges

E. You cannot conclude anything

Solution: Since $f(x) \geq g(x) > 0$ for all $x > 1$ and $\int_1^\infty g(x)\,dx$ diverges, the Direct Comparison Theorem implies that

$$\int_1^\infty f(x)\,dx$$

also diverges. Therefore the correct choice is **D**.

4. Free response question.

Solution: All of these examples are from the Fall 2022 exam. The solutions are posted on Canvas under "Exams."