

1. (2 points) Which of the following is the result of a valid u -substitution for the following integral?

$$\int_0^{\pi/6} \sin^3(x) \cos^4(x) dx$$

(a) $\int_1^{\sqrt{3}/2} (u^6 - u^4) du$

(b) $\int_0^{1/2} u^3(1-u^2)^2 du$

(c) $\int_1^{\sqrt{3}/2} u^4(1-u)^2 du$

(d) $\int_0^{\sqrt{3}/2} u^3(1-u^4) du$

(e) $\int_0^{\pi/6} u^3 u^4 du$

$\int_0^{\pi/6} \sin^2(x) \cos^4(x) \cdot \sin(x) dx$

$\int_0^{\pi/6} (1-\cos^2(x)) \cos^4(x) \cdot \sin(x) dx$

$u = \cos(x) \quad du = -\sin(x)$

$-\int_{\sqrt{3}/2}^1 (1-u^2) u^4 du$

$\int_1^{\sqrt{3}/2} u^6 - u^4 du$

2. (2 points) Which of the following is the result of a valid u -substitution for the following integral?

$$\int_0^{\pi/4} \sec^4(x) \tan^4(x) dx$$

(a) $\int_0^1 u^4(1+u^2) du$

(b) $\int_0^1 u^4(1-u^2) du$

(c) $\int_0^1 u^3(1+u^2) du$

(d) $\int_1^{\sqrt{2}} (u^2-1)^2 du$

(e) $\int_0^{\pi/4} u^4 u^2 du$

$\int_0^{\pi/4} \sec^2(x) \tan^4(x) \cdot \sec^2(x) dx$

$\int_0^{\pi/4} (1+\tan^2(x)) \tan^4(x) \cdot \sec^2(x) dx$

let $u = \tan(x) \quad du = \sec^2(x)$

$\int_0^1 (1+u^2) u^4 du$

3. (2 points) Which substitution is appropriate for computing the following integral?

$$\int \frac{\sqrt{x^2+16}}{x^2} dx$$

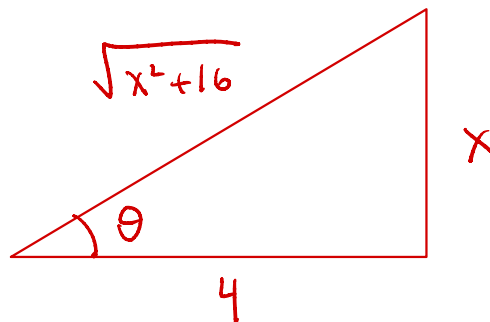
(a) $x = 4 \sec \theta$

(b) $x = 4 \tan \theta$

(c) $x = 4 \sin \theta$

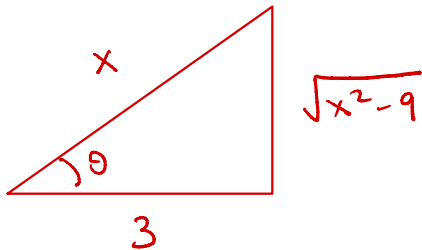
(d) $x = 4 \csc \theta$

(e) $x = 16 \sec \theta$



4. (4 points) Compute the following indefinite integral.

$$\int \frac{1}{x^2 \sqrt{x^2 - 9}} dx$$



$$\text{Let } x = 3 \sec \theta$$

$$\text{Then } dx = 3 \sec \theta \tan \theta d\theta$$

$$\text{From the triangle, } \sqrt{x^2 - 9} = 3 \tan \theta$$

$$\int \frac{1}{x^2 \sqrt{x^2 - 9}} dx = \int \frac{1}{9 \sec^2 \theta \cdot 3 \tan \theta} \cdot 3 \sec \theta \tan \theta d\theta$$

$$= \int \frac{1}{9 \sec \theta} d\theta$$

$$= \frac{1}{9} \int \cos \theta d\theta$$

$$= \frac{1}{9} \sin \theta + C$$

$$\text{From the triangle, } \sin \theta = \frac{\sqrt{x^2 - 9}}{x}$$

$$= \frac{1}{9} \cdot \frac{\sqrt{x^2 - 9}}{x} + C$$