Math 2300
Spring 2026
Quiz 12

Name: Champ

1. (2 points) Determine the value of the following series, if it converges.

$$\sum_{n=1}^{\infty} \frac{5^{n+1}}{3^{2n-1}}$$

   (a) $\frac{25}{3}$
   (b) $\frac{75}{4}$
   (c) $\frac{45}{4}$
   (d) $\frac{75}{8}$
   (e) The series diverges.

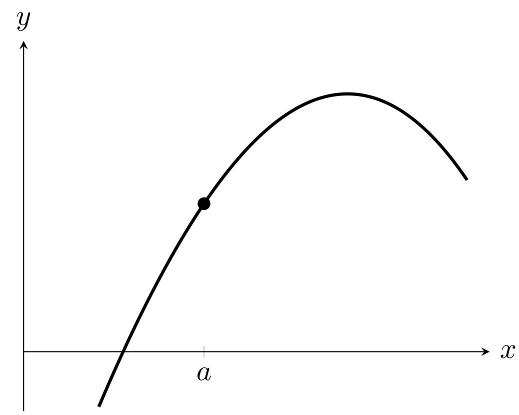
$$\sum_{n=1}^{\infty} \frac{5^{n+1}}{3^{2n-1}} = \sum_{n=1}^{\infty} \frac{5\cdot 5^n}{3^{2n}\cdot 1/3} = \sum_{n=1}^{\infty} 15\cdot \frac{5^n}{(3^2)^n} = \sum_{n=1}^{\infty} 15\cdot\left(\frac{5}{9}\right)^n$$

Converges to $\frac{\text{first term}}{1-r} = \frac{25/3}{1-5/9} = \frac{25/3}{4/9} = \frac{25}{3}\cdot\frac{9}{4} = \frac{75}{4}$

2. (2 points) The graph of $y = f(x)$ is shown below. Which of the following could be the second-degree Taylor polynomial for f centered at $x = a$?

   (a) $(x-a)^2 - 3(x-a) + 2$
   (b) $(x-a)^2 + 3(x-a) + 2$
   (c) $-(x-a)^2 + 3(x-a) + 2$
   (d) $-(x-a)^2 - 3(x-a) + 2$
   (e) $-(x-a)^2 + 3(x-a) - 2$

Concave down
increasing
positive

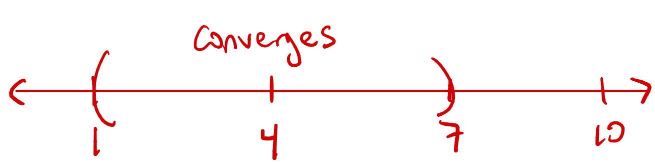


3. (2 points) You are told that the power series

$$\sum_{n=1}^{\infty} a_n (x-4)^n$$

converges at $x = 1$ and diverges at $x = 10$. At which of the following values of x is the series guaranteed to converge?

   (a) $x = 9$
   (b) $x = 7$
   (c) $x = 0$
   (d) $x = 6$
   (e) $x = -2$



4. (4 points) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n(3x)^n}{n^2}.$$

Ratio Test:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}(3x)^{n+1}}{(n+1)^2}\cdot\frac{n^2}{(-1)^n(3x)^n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}}{(-1)^n}\cdot\frac{n^2}{(n+1)^2}\cdot\frac{(3x)^{n+1}}{(3x)^n}\right|$$

$$= \lim_{n\to\infty}\frac{n^2}{(n+1)^2}\cdot|3x| = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^2\cdot|3x| = |3x|$$

The series converges for $|3x| < 1 \Rightarrow -1 < 3x < 1 \Rightarrow -\frac{1}{3} < x < \frac{1}{3}$

Check Endpoints:

- At $x = -1/3$:

$$\sum_{n=1}^{\infty}\frac{(-1)^n(-1)^n}{n^2} = \sum_{n=1}^{\infty}\frac{1}{n^2}$$ is a convergent p-series $(p = 2 > 1)$

- At $x = 1/3$

$$\sum_{n=1}^{\infty}\frac{(-1)^n(1)^n}{n^2} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$ converges absolutely (and hence converges)

Since $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is a p-series $(p = 2 > 1)$

$$\boxed{\text{Interval of Convergence: } [-1/3, 1/3]}$$