Math 2300
Spring 2026
Quiz 11

Name: Champ

1. (2 points) What does the **Ratio Test** tell us about the following series?

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{n^2}{(n+1)^2} = 1$$

   (a) The Ratio Test tells us that the series converges absolutely.

   (b) The Ratio Test tells us that the series converges conditionally.

   (c) The Ratio Test tells us that the series diverges.

   (d) The Ratio Test is inconclusive.

2. (2 points) Let

$$S = \sum_{n=1}^{\infty} \frac{1}{n^3}.$$

If $R_2 = S - S_2$ is the remainder after the 2nd partial sum, which of the following must be true?

   (a) $\frac{1}{18} \le R_2 \le \frac{1}{8}$

   (b) $\frac{1}{8} \le R_2 \le \frac{1}{4}$

   (c) $0 \le R_2 \le \frac{1}{18}$

   (d) $\frac{1}{27} \le R_2 \le \frac{1}{18}$

   (e) $R_2 = \frac{1}{8}$

$$\int_3^{\infty} \frac{1}{x^3}\,dx \le R_2 \le \int_2^{\infty} \frac{1}{x^3}\,dx$$

$$\lim_{t\to\infty} \left[-\frac{1}{2x^2} \right]_3^t \le R_2 \le \lim_{t\to\infty} \left[-\frac{1}{2x^2} \right]_2^t$$

$$\frac{1}{18} \le R_2 \le \frac{1}{8}$$

3. (2 points) Let

$$S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{(3n-1)^2}.$$

If S_5 is used to approximate S, what is the largest possible value of $|S - S_5|$? Use the Alternating Series Estimation Theorem, and do not simplify your answer.

$$\boxed{|S - S_5| \le b_6 = \frac{1}{17^2}}$$

4. (4 points) Let f be the function given by $f(x) = \ln x$.

Center is a

(a) (3 points) Find the third-degree Taylor polynomial for f centered at $x = 1$.

① $P_3(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3$

② $f(x) = \ln(x)$ $\quad f(1) = 0$

$f'(x) = \frac{1}{x} = x^{-1}$ $\quad f'(1) = 1$

$f''(x) = -x^{-2} = -\frac{1}{x^2}$ $\quad f''(1) = -1$

$f'''(x) = 2x^{-3} = \frac{2}{x^3}$ $\quad f'''(1) = 2$

③ $P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

(b) (1 point) Use the Taylor polynomial found in part (a) to approximate $\ln(1.1)$. You do not need to simplify your answer.

$\ln(1.1) = f(1.1) \approx P_3(1.1) = 0.1 - \frac{1}{2}(0.1)^2 + \frac{1}{3}(0.1)^3$