Math 2300
Spring 2026
Quiz 10

Name: Champ

1. (5 points) Consider the series

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt[3]{n}}.$$

Does the series converge absolutely, converge conditionally, or diverge? Fully justify your answer by stating any test(s) used, verifying that the series satisfies the conditions of the test, and clearly explaining each step leading to your conclusion.

Solution: We first check for **absolute convergence**. Consider

$$\sum_{n=1}^{\infty}\left|(-1)^n\frac{1}{\sqrt[3]{n}}\right| = \sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}} = \sum_{n=1}^{\infty}\frac{1}{n^{1/3}}.$$

This is a p-series with

$$p = \frac{1}{3}.$$

Since $p \leq 1$, the p-series diverges. Therefore, the given series does *not* converge absolutely.

Next, we check whether the series converges by the **Alternating Series Test**.

1. b_n is decreasing, because $n^{1/3}$ increases as n increases, so $\frac{1}{n^{1/3}}$ decreases.
2. $\lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{1}{n^{1/3}} = 0.$

Therefore, the series converges by the Alternating Series Test. Because the series converges, but the corresponding absolute value series diverges, the series

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt[3]{n}}$$

converges conditionally.

2. (5 points) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{2^n}{(2n)!}$$

converges or diverges. Fully justify your answer by stating any test(s) used, verifying that the series satisfies the conditions of the test, and clearly explaining each step leading to your conclusion.

Solution: We will use the **Ratio Test**. Let

$$a_n = \frac{2^n}{(2n)!}.$$

Compute

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{2^{n+1}}{(2n+2)!}}{\frac{2^n}{(2n)!}} \right| = \lim_{n\to\infty} \left(\frac{2^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{2^n} \right)$$

$$= \lim_{n\to\infty} \left(2 \cdot \frac{(2n)!}{(2n+2)!} \right)$$

$$= \lim_{n\to\infty} \left(2 \cdot \frac{(2n)!}{(2n+2)(2n+1)(2n)!} \right)$$

$$= \lim_{n\to\infty} \frac{2}{(2n+2)(2n+1)}$$

$$= 0.$$

Since $L = 0 < 1$, the series $\sum_{n=1}^{\infty} \frac{2^n}{(2n)!}$ **converges** by the Ratio Test.