Math 2300
Spring 2026
Quiz 1

Name: Champ

1. (2 points) Evaluate $\int_0^{\pi/2} \sin x \sqrt{1 - \cos x}\, dx$.

(a) $-\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) 1

(e) $\frac{4}{3}$

Let $u = 1 - \cos x$. Then $du = \sin x\, dx$.

When $x = 0$, $u = 0$. When $x = \pi/2$, $u = 1$.

$$\int_{x=0}^{x=\pi/2} \sin x \cdot \sqrt{1-\cos x}\, dx = \int_{u=0}^{u=1} \sqrt{u}\, du = \left[\frac{2}{3} u^{3/2}\right]_{u=0}^{u=1} = \frac{2}{3}$$

2. (2 points) Suppose that $\int_1^9 f(x)\, dx = 12$. Compute $\int_0^{2\sqrt{2}} 4x\, f(x^2+1)\, dx$.

(a) 3

(b) 6

(c) 12

(d) 24

(e) 48

Let $u = x^2 + 1$. Then $du = 2x \cdot dx \Rightarrow 4x \cdot dx = 2du$

When $x = 0$, $u = 1$. When $x = 2\sqrt{2}$, $u = 9$.

$$\int_0^{2\sqrt{2}} 4x\, f(x^2+1)\, dx = \int_1^9 2 f(u)\, du = 2 \cdot \int_1^9 f(u)\, du = 2 \cdot 12 = 24$$

3. (2 points) Compute $\int_1^2 x^3 \ln x\, dx$. You may use the fact that $\int_1^2 x^3\, dx = \frac{15}{4}$.

(a) $4\ln 2 - \frac{15}{16}$

(b) $4\ln 2 - 1$

(c) $16\ln 2 - \frac{15}{4}$

(d) $2\ln 2 - \frac{7}{16}$

(e) $4\ln 2 + \frac{15}{16}$

Let $u = \ln x$ and $dv = x^3\, dx$. Then $du = \frac{1}{x} dx$ and $v = \frac{1}{4} x^4$.

$$\int_1^2 x^3 \ln x\, dx = \left[\ln x \cdot \frac{1}{4} x^4\right]_1^2 - \int_1^2 \frac{1}{4} x^4 \cdot \frac{1}{x}\, dx$$

$$= \left[\ln(2) \cdot 4 - \ln(1) \cdot \frac{1}{4}\right] - \frac{1}{4} \int_1^2 x^3\, dx$$

$$= \left[4\ln(2) - 0\right] - \frac{1}{4} \cdot \frac{15}{4}$$

$$= 4\ln(2) - \frac{15}{16}$$

4. (4 points) Compute the following indefinite integral.

$$\int e^{2x} \sin x \, dx$$

Let $u = \sin x$ $\qquad v = \frac{1}{2}e^{2x}$

$du = \cos x \, dx$ $\qquad dv = e^{2x} dx$

Then

$$\int e^{2x} \sin x \, dx = \frac{1}{2}e^{2x} \sin x - \int \frac{1}{2}e^{2x} \cos x \, dx$$
$$= \frac{1}{2}e^{2x} \sin x - \frac{1}{2}\int e^{2x} \cos x \, dx$$

Let $u = \cos x$ $\qquad v = \frac{1}{2}e^{2x}$

$du = -\sin x \, dx$ $\qquad dv = e^{2x} dx$

$$\int e^{2x} \cos x \, dx = \frac{1}{2}e^{2x} \cos x - \int \frac{1}{2}e^{2x} (-\sin x) \, dx$$
$$= \frac{1}{2}e^{2x} \cos x + \frac{1}{2}\int e^{2x} \sin x \, dx$$

In total,

$$\int e^{2x} \sin x \, dx = \frac{1}{2}e^{2x} \sin x - \frac{1}{2}\left(\frac{1}{2}e^{2x} \cos x + \frac{1}{2}\int e^{2x} \sin x \, dx\right)$$
$$= \frac{1}{2}e^{2x} \sin x - \frac{1}{4}e^{2x} \cos x - \frac{1}{4}\int e^{2x} \sin x \, dx$$

$$\Rightarrow \quad \frac{5}{4}\int e^{2x} \sin x \, dx = \frac{1}{2}e^{2x} \sin x - \frac{1}{4}e^{2x} \cos x$$

$$\Rightarrow \quad \int e^{2x} \sin x \, dx = \frac{2}{5}e^{2x} \sin x - \frac{1}{5}e^{2x} \cos x + C$$