Quiz 1 Outline

Format. This quiz has **3 multiple-choice** questions and **1 free-response** question.

1. (2 points) Compute an integral using u-substitution.

Example: $\int_0^2 xe^{x^2}\,dx =$

A. $\frac{1}{2}e - \frac{1}{2}$

B. $2e - 2$

C. $4e^2 - 4$

D. $\frac{1}{2}e^4 - \frac{1}{2}$

E. $2e^4 - 2$

Spring 2023 Exam 1 #1

2. (2 points) Use u-substitution with a function $f(x)$ and a known integral.

Example: Suppose that $\int_2^{14} f(x)\,dx = 15$. Compute the value of $\int_0^2 12x\,f(3x^2+2)\,dx$.

A. 5

B. 15

C. 30

D. 45

Fall 2022 Exam 1 #3

3. (2 points) Use integration by parts to evaluate an integral. Could be either an indefinite integral or a definite integral, possibly using the value of a known integral.

Example: Compute $\int_0^1 x^3e^x\,dx$. You may use the fact that $3\int_0^1 x^2e^x\,dx = 3e - 6$.

A. $3e - 6$

B. $2e - 4$

C. $2 - 3e$

D. $10 - 2e$

E. $6 - 2e$

Spring 2023 Exam 1 #4

4. (4 points) Boomerang integral.

Example: Compute the following indefinite integral.

$$\int e^{2x} \cos x \, dx$$

Fall 2022 Exam 1 #8

1. $\displaystyle\int_0^2 xe^{x^2}\,dx =$

A. $\dfrac{1}{2}e - \dfrac{1}{2}$

B. $2e - 2$

C. $4e^2 - 4$

D. $\dfrac{1}{2}e^4 - \dfrac{1}{2}$

E. $2e^4 - 2$

Solution: Let $u = x^2$. Then $du = 2x\,dx$, so $x\,dx = \frac{1}{2}\,du$. Change the bounds:

$$x = 0 \Rightarrow u = 0, \qquad x = 2 \Rightarrow u = 4.$$

Thus

$$\begin{aligned} \int_0^2 xe^{x^2}\,dx &= \frac{1}{2}\int_0^4 e^u\,du \\ &= \frac{1}{2}\Big[e^u\Big]_0^4 \\ &= \frac{1}{2}\left(e^4 - 1\right) \\ &= \frac{1}{2}e^4 - \frac{1}{2}. \end{aligned}$$

Answer: $\boxed{\text{D}}$

2. Suppose that $\int_2^{14} f(x)\,dx = 15$. Compute the value of $\int_0^2 12x\,f(3x^2+2)\,dx$.

A. 5

B. 15

C. 30

D. 45

Solution: Let $u = 3x^2 + 2$. Then $du = 6x\,dx$, so $12x\,dx = 2\,du$. Also,

$$x = 0 \Rightarrow u = 2, \qquad x = 2 \Rightarrow u = 14.$$

Thus

$$\begin{aligned}\int_0^2 12x\,f(3x^2+2)\,dx &= \int_2^{14} 2\,f(u)\,du \\ &= 2\int_2^{14} f(u)\,du \\ &= 2 \cdot 15 \\ &= 30.\end{aligned}$$

Answer: $\boxed{\text{C}}$

3. Compute $\int_0^1 x^3 e^x \, dx$. You may use the fact that

$$3\int_0^1 x^2 e^x \, dx = 3e - 6.$$

A. $3e - 6$

B. $2e - 4$

C. $2 - 3e$

D. $10 - 2e$

E. $6 - 2e$

Solution: Use integration by parts with $u = x^3$, $dv = e^x \, dx$. Then $du = 3x^2 \, dx$, $v = e^x$.

$$\begin{aligned}\int_0^1 x^3 e^x \, dx &= \left[x^3 e^x\right]_0^1 - \int_0^1 3x^2 e^x \, dx \\ &= e - \left(3\int_0^1 x^2 e^x \, dx\right).\end{aligned}$$

Given $3\int_0^1 x^2 e^x \, dx = 3e - 6$, we get

$$\int_0^1 x^3 e^x \, dx = e - (3e - 6) = 6 - 2e.$$

Answer: $\boxed{\text{E}}$

4. Compute the indefinite integral

$$\int e^{2x} \cos x \, dx.$$

Solution: Let

$$A = \int e^{2x} \cos x \, dx.$$

Integrate by parts with $u = \cos x$, $dv = e^{2x}\, dx$. Then $du = -\sin x \, dx$ and $v = \frac{1}{2}e^{2x}$, so

$$A = \frac{1}{2}e^{2x} \cos x + \frac{1}{2} \int e^{2x} \sin x \, dx.$$

Now define

$$B = \int e^{2x} \sin x \, dx.$$

Integrate by parts with $u = \sin x$, $dv = e^{2x}\, dx$. Then $du = \cos x \, dx$, $v = \frac{1}{2}e^{2x}$, giving

$$\begin{aligned} B &= \frac{1}{2}e^{2x} \sin x - \frac{1}{2} \int e^{2x} \cos x \, dx \\ &= \frac{1}{2}e^{2x} \sin x - \frac{1}{2}A. \end{aligned}$$

Substitute this into the earlier equation for A:

$$\begin{aligned} A &= \frac{1}{2}e^{2x} \cos x + \frac{1}{2}B \\ &= \frac{1}{2}e^{2x} \cos x + \frac{1}{2}\left(\frac{1}{2}e^{2x} \sin x - \frac{1}{2}A\right) \\ &= \frac{1}{2}e^{2x} \cos x + \frac{1}{4}e^{2x} \sin x - \frac{1}{4}A. \end{aligned}$$

Solve for A:

$$\frac{5}{4}A = \frac{1}{2}e^{2x} \cos x + \frac{1}{4}e^{2x} \sin x \quad \Longrightarrow \quad A = \frac{2}{5}e^{2x} \cos x + \frac{1}{5}e^{2x} \sin x$$

$$\boxed{\int e^{2x} \cos x \, dx = \frac{e^{2x}}{5}\big(2\cos x + \sin x\big) + C.}$$