

Math 2300: Final Exam Practice

1. Evaluate $\int_0^1 xe^{2x} dx$.
2. Evaluate $\int \frac{x}{\sqrt{1+x^2}} dx$.
3. Evaluate $\int \sin^5(x) \cos^2(x) dx$.
4. Evaluate $\int \sec^3(x) \tan^3(x) dx$.
5. Evaluate $\int \frac{1}{x^2 \sqrt{x^2 - 16}} dx$.
6. Evaluate $\int \frac{5x - 1}{x^2 - 2x - 3} dx$.
7. The general solution to a differential equation is $y = Ce^{3x} - x - \frac{1}{3}$. Find the particular solution satisfying the initial condition $y(0) = 2$.
8. Solve the differential equation $y' = y^2$, with $y(0) = 1$.
9. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = 4$ about the x -axis.
10. Find the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = x^2$ about the y -axis.
11. A tank has the shape of an inverted circular cone with height 9 m and base radius 4 m. It is filled with water to a height of 5 m (measured upward from the tip). How much work is required to pump all of the water to the top rim of the tank? Set up, but do not evaluate, an integral for the work required. Assume the density of water is 1000 kg/m^3 and take $g = 9.8 \text{ m/s}^2$.
12. Compute the center of mass of the region bounded by $y = \sin x$, $y = 0$, $x = 0$, and $x = \pi$.
13. Compute the equation of the tangent line to the parametric curve $x = \ln(t)$, $y = t^2$ at $t = 1$.
14. For the parametric curve
$$x = t^3 - 3t, \quad y = t^4 - 4t^2,$$
find all values of t at which the curve has a horizontal tangent line.

15. Find the arc length of the curve $x = t^2$, $y = t^3$ on the interval $0 \leq t \leq 1$.
16. Compute the area inside one petal of the rose $r = 2 \sin(3\theta)$.
17. Find the area of the polar region that lies outside $r = \sqrt{3}$ and inside $r = 2 \cos \theta$.
18. Find the sum of the series $\sum_{n=1}^{\infty} \left(\frac{1}{3n} - \frac{1}{3(n+4)} \right)$.
19. Evaluate the series $\sum_{n=3}^{\infty} \frac{2^n}{5^{n+2}}$.
20. Determine whether the series $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$ converges or diverges.
21. Determine whether the series $\sum_{n=1}^{\infty} \frac{\arcsin(1/n)}{n^3}$ converges or diverges. Justify your answer.
22. Find a power series representation for $x \cos(3x^2)$.
23. Evaluate the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)3^{2n+1}}$.
24. Find a power series representation for $\int \frac{x^2}{1+x^3} dt$.
25. Find the Taylor series for $f(x) = x \ln(x)$ centered at $a = 4$.
26. Find the Taylor series for $f(x) = \frac{1}{x}$ centered at $a = 3$. Also, determine its interval of convergence.
27. Use Taylor's Inequality to determine the minimum degree n such that the Taylor polynomial $T_n(x)$ for e^x , centered at $a = 2$, approximates e^3 to within 0.01.
28. Use Taylor's Inequality to determine the interval of x -values such that the third-degree Maclaurin polynomial $T_3(x)$ for $\sin x$ approximates $\sin x$ to within $\frac{1}{384}$.