Math 2300: Final Exam Practice

1. Evaluate the integral: $\displaystyle\int \frac{2x}{x^2+5}\,dx$

2. Evaluate the integral: $\displaystyle\int \ln(x)\,dx$

3. Evaluate the integral: $\displaystyle\int \frac{x+1}{(x-1)(x+2)}\,dx$ using partial fraction decomposition.

4. Evaluate the integral: $\displaystyle\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$

5. Evaluate the integral: $\displaystyle\int xe^{x^2}\,dx$ using an appropriate method.

6. Find the slope $\frac{dy}{dx}$ of the parametric curve at $t = \pi/6$, where $x(t) = \sin(2t)$ and $y(t) = \cos(t)$

7. Find the average value of the function $f(x) = \sqrt{x}$ on the interval $[1, 4]$.

8. Find the volume of the solid with base bounded by $y = x^2$ and $y = 4$, where cross-sections perpendicular to the x-axis are semicircles.

9. Set up an integral representing the volume of the solid formed by rotating the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the x-axis.

10. Set up, but do not evaluate, an integral representing the area enclosed by one loop of the curve:

$$r(\theta) = 3\sin(2\theta)$$

11. Find the exact sum of the series:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

by recognizing it as a telescoping series.

12. Determine whether the following series converges or diverges: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n+2}$

13. Find the radius of convergence and interval of convergence for the series:

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n2^n}$$

14. Find the first four nonzero terms of the Taylor series for $f(x) = e^{-x^2}$ centered at $a = 0$.

15. Find the third-degree Taylor polynomial centered at $a = 0$ for $f(x) = \sin(2x)$.

16. Find the third degree Taylor polynomial for $f(x) = \cos x$ centered at $a = \pi/6$. Use Taylor's Inequality to give an upper bound on the error if this approximation is used on the interval $0 < x < \pi/3$.

17. Solve the differential equation:

$$\frac{dy}{dx} = xy^2, \quad y(0) = 1$$