Math 2300: Midterm 3 Practice

1. Evaluate $\lim_{n\to\infty} \dfrac{\sqrt{n^2+\cos n}}{n}$.

2. Suppose $S_n = \sum_{k=1}^{n} a_k = \dfrac{n^2}{\ln(n+1)+3n}$. Determine whether $\sum_{n=1}^{\infty} a_n$ converges and, if so, to what value.

3. Suppose $S_n = \sum_{k=1}^{n} a_k = \dfrac{4n^2}{\ln(n+1)+4n^2}$. Determine whether $\sum_{n=1}^{\infty} a_n$ converges and, if so, to what value.

4. Suppose $S_n = \sum_{k=1}^{n} a_k = \dfrac{4n^2+1}{2n^2+5n+1}$. Determine whether $\sum_{n=1}^{\infty} a_n$ converges and, if so, to what value.

5. If $\sum a_n = \pi$ and $\sum b_n = 4$, find $\sum(2a_n - 3b_n)$.

6. Find the sum of the geometric series: $\sum_{n=2}^{\infty} \dfrac{6}{4^n}$.

7. Determine whether the series $\sum_{n=2}^{\infty} \dfrac{n^4+3}{n(n+1)^2}$ converges. If it converges, find its sum.

8. Determine whether the series $\sum_{n=1}^{\infty} \dfrac{4^n}{5^n-1}$ converges.

9. Let $a_n = \dfrac{n^2}{2^n}$. Use the Ratio Test to determine whether $\sum a_n$ converges or diverges.

10. Let $a_n = \dfrac{n!}{2^n}$. Use the Ratio Test to determine whether $\sum a_n$ converges or diverges.

11. Use the Ratio Test to determine for which values of $c > 0$ the series $\displaystyle\sum_{n=1}^{\infty} \frac{c^n}{n}$ converges.

12. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n^3 + \cos n}$ converges or diverges.

13. Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n^3)}{n^3 + 1}$ converges absolutely, conditionally, or diverges.

14. Determine whether $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n}{n^2 - 4}$ converges absolutely, conditionally, or diverges.

15. Use the Alternating Series Remainder Theorem to estimate the error when approximating $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^3}$ using the first 4 terms.

16. Use the Integral Test to show that $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2}{(n^3+1)^3}$ converges. Remember to verify that the corresponding function $\displaystyle\sum_{n=1}^{\infty} \frac{3x^2}{(x^3+1)^3}$ is positive, continuous, and decreasing for $x \geq 1$. Estimate the error when approximating the series with the first 4 terms.

17. Use the Integral Test to show that $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln(n)^2}$ converges. Remember to verify that the corresponding function $\displaystyle\sum_{n=2}^{\infty} \frac{1}{x \ln(x)^2}$ is positive, continuous, and decreasing for $x \geq 2$. Estimate the error when approximating the series with the first 5 terms.

18. If the third degree Taylor polynomial of $f(x)$ about $x = 4$ is

$$T_3(x) = 5 + 4(x-4) - 3(x-4)^2 - 2(x-4)^3$$

is $f(x)$ increasing or decreasing at $x = 4$? Is $f(x)$ concave up or concave down at $x = 4$?