Review of Calculus I

This document provides a review of essential Calculus I topics that are foundational for success in Calculus II.

1 Limits and Continuity

- **Definition of a limit:** $\lim_{x\to c} f(x) = L$. This means that as x approaches c, $f(x)$ gets arbitrarily close to L.
- **One-sided limits:** $\lim_{x\to c^+} f(x)$ (approaching from the right) and $\lim_{x\to c^-} f(x)$ (approaching from the left).
- **Limit laws:**
  - $\lim_{x\to c}[f(x) + g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$.
  - $\lim_{x\to c}[f(x)g(x)] = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$.
  - $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$, provided $\lim_{x\to c} g(x) \neq 0$.
- **Techniques for computing limits:**
  - Direct substitution.
  - Factoring and simplifying.
  - Rationalizing the numerator or denominator.
  - L'Hôpital's Rule for indeterminate forms like $\frac{0}{0}$ or $\frac{\infty}{\infty}$.
- **Continuity:** A function $f(x)$ is continuous at $x = c$ if:
  1. $\lim_{x\to c} f(x)$ exists.
  2. $f(c)$ is defined.
  3. $\lim_{x\to c} f(x) = f(c)$.
- **Types of discontinuities:**
  - Removable discontinuity: Limit exists, but $f(c)$ is undefined or not equal to the limit.
  - Jump discontinuity: Left-hand and right-hand limits are not equal.
  - Infinite discontinuity: Limit approaches ∞ or $-\infty$.

2 Derivatives

Key Concepts

- **Definition:** $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.
- **Differentiation rules:**
  1. **Power Rule:** $\frac{d}{dx}[x^n] = nx^{n-1}$.
  2. **Product Rule:** $\frac{d}{dx}[uv] = u'v + uv'$.
  3. **Quotient Rule:** $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v-uv'}{v^2}$.
  4. **Chain Rule:** $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$.
- **Applications:**
  - Tangent lines: The slope of the tangent to $f(x)$ at $x = a$ is $f'(a)$.
  - Velocity and acceleration: $v(t) = s'(t),\ a(t) = v'(t)$.
  - Optimization problems: Finding maxima or minima using $f'(x) = 0$.
  - Related rates: Solving problems involving rates of change of related variables.

General Derivative Rules

Rule	**Formula**
Power Rule	$\frac{d}{dx}x^n = nx^{n-1}$
Product Rule	$\frac{d}{dx}[uv] = u'v + uv'$
Quotient Rule	$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v-uv'}{v^2}$
Chain Rule	$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$
Exponential Rule	$\frac{d}{dx}e^x = e^x$
Logarithmic Rule	$\frac{d}{dx}\ln x = \frac{1}{x}$

Trigonometric Derivatives

Function	**Derivative**
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$

3 Integrals

- **Definition:** $\int_a^b f(x)\,dx$ computes the net signed area under the curve $f(x)$ between $x = a$ and $x = b$. The area is positive when the function is above the x-axis and negative when below.
- **Fundamental Theorem of Calculus:**
  - **Part 1:** If $F(x)$ is an antiderivative of $f(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.
  - **Part 2:** $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.
- **Basic Properties:**
  - Linearity: $\int_a^b [c_1 f(x) + c_2 g(x)]\,dx = c_1 \int_a^b f(x)\,dx + c_2 \int_a^b g(x)\,dx$.
  - Additivity: $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$.
  - Reversal of Limits: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$.

Common Integration Formulas

Function	Integral
k	$\int k\,dx = kx + C$
$x^n,\ n \neq -1$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$
e^x	$\int e^x\,dx = e^x + C$
$\frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\sin x$	$\int \sin x\,dx = -\cos x + C$
$\cos x$	$\int \cos x\,dx = \sin x + C$
$\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
$\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$