9.3 Separable Differential Equations

Definition. A differential equation is **separable** if it can be written in the form

$$\frac{dy}{dx} = g(x)h(y)$$

This allows separation of variables:

$$\frac{1}{h(y)}\,dy = g(x)\,dx$$

Steps for Solving a Separable Differential Equation

1. Rewrite in the form $\frac{dy}{dx} = g(x)h(y)$.
2. Separate variables: $\frac{1}{h(y)}\,dy = g(x)\,dx$.
3. Integrate both sides.
4. Solve explicitly for y, if possible.
5. Apply any initial condition to determine the constant C.

Remark. If the integral involves $\int \frac{1}{y}\,dy$, the solution includes a logarithm:

$$\int \frac{1}{y}\,dy = \ln|y| + C$$

Be careful with absolute values when solving for y.

Multiple Choice Practice

1. Which of the following differential equations is separable?

   (A) $\frac{dy}{dx} = x^2 + y^2$

   (B) $\frac{dy}{dx} = e^x + y$

   (C) $\frac{dy}{dx} = xy$

   (D) $\frac{dy}{dx} = \tan(x + y)$

   (E) $\frac{dy}{dx} = \ln(x + y)$

2. Which of the following is NOT separable?

   (A) $\frac{dy}{dx} = \frac{x}{1 + y^2}$

   (B) $\frac{dy}{dx} = x(1 + y^2)$

   (C) $\frac{dy}{dx} = x + y$

   (D) $\frac{dy}{dx} = \frac{y}{x}$

   (E) $\frac{dy}{dx} = \frac{y^2}{x}$

3. Solve for the general solution to $\dfrac{dy}{dx} = x^2 y$.

(A) $y = Ce^{x^2}$
(B) $y = Cx^2$
(C) $y = Ce^{x^3/3}$
(D) $y = Cxe^x$
(E) $y = Cx^3 + 1$

4. Solve for the general solution to $\dfrac{dy}{dx} = \dfrac{3y}{x}$.

(A) $y = Cx^3$
(B) $y = Cx$
(C) $y = Cx^2$
(D) $y = Ce^{3x}$
(E) $y = Cx^{-3}$

5. Find the particular solution to $\dfrac{dy}{dx} = y^2$ with initial condition $y(0) = 2$.

(A) $y = \dfrac{1}{2 - x}$
(B) $y = \dfrac{1}{x + 2}$
(C) $y = 2 + x$
(D) $y = 2 - x$
(E) $y = \dfrac{1}{x - 2}$

6. Find the particular solution to $\dfrac{dy}{dx} = (1 - x^2)y$ with $y(0) = 3$.

(A) $y = 3e^{-x+x^3/3}$
(B) $y = 3e^{x-x^3/3}$
(C) $y = 3e^{x^2/2}$
(D) $y = 3e^{x-x^2}$
(E) $y = 3e^{x^3-x}$

7. Solve the separable differential equation $\dfrac{dy}{dx} = \dfrac{2y}{x}$.

(A) $y = Cx$
(B) $y = Cx^2$
(C) $y = Cx^3$
(D) $y = Cx^4$
(E) $y = C\ln(x)$

8. Solve the separable differential equation $\dfrac{dy}{dx} = \dfrac{x^2}{y}$.

(A) $y^2 = \dfrac{2x^3}{3} + C$

(B) $y = Cx^2$

(C) $y = \dfrac{x^3}{3}$

(D) $y^2 = x^2 + C$

(E) $y = e^{x^2}$

Free Response Practice

1. Solve the differential equation $y' = xy$ by finding the general solution.
2. Solve the differential equation $y' = \dfrac{2y}{x}$ by finding the general solution.
3. Solve the differential equation $y' = x(1 + y^2)$ by finding the general solution.
4. Solve the differential equation: $y' = y^2$ with the initial condition $y(0) = 1$.
5. Solve the differential equation: $y' = (1 - x^2)y$ with $y(0) = 2$.
6. Solve the differential equation: $y' = x^2(1 + y)$ given that $y(1) = 0$.
7. Solve the differential equation: $y' = \dfrac{y}{x}$ and leave your answer in implicit form if necessary.
8. Solve the differential equation: $y' = \dfrac{x}{y}$ and leave your answer in implicit form if necessary.
9. Solve the differential equation: $y' = \dfrac{2y}{x + 1}$ and leave your answer in implicit form if necessary.