9.1 Differential Equations

Definition. A **differential equation** is an equation that relates an unknown function to one or more of its derivatives.

Key vocabulary.

- **Order:** the highest-order derivative that appears.
- **General solution:** the full family of solutions (typically containing one or more arbitrary constants).
- **Particular solution:** one specific member of that family.
- **Initial-value problem (IVP):** a differential equation together with an initial condition such as $y(t_0) = y_0$, used to select the particular solution.

Remark. To verify that $y = f(x)$ satisfies a differential equation:

1. Compute the derivatives that appear in the equation.
2. Substitute $y = f(x)$ and those derivatives into the differential equation.
3. Simplify. If the left and right sides match for all x in the interval, then f is a solution.

Multiple Choice Practice

1. Verify whether $y = e^{2x}$ is a solution to $y' = 2y$.

   (A) Yes, it satisfies the differential equation.
   (B) No, it does not satisfy the differential equation.

2. Is $y = x^2 + 1$ a solution to the differential equation $y' = 2x$?

   (A) Yes
   (B) No

3. Which of the following functions satisfy $y' = 3y$? (Select all that apply.)

   (a) $y = e^{3x}$
   (b) $y = 2e^{3x}$
   (c) $y = e^{-3x}$
   (d) $y = 3e^{x}$

4. Which of the following satisfy $y'' + y = 0$? (Select all that apply).

   (a) $y = \sin(x)$
   (b) $y = \cos(x)$
   (c) $y = e^{x}$
   (d) $y = \sin(x) + \cos(x)$

5. Solve the differential equation $y' = 6x$.

   (A) $y = 6x + C$
   (B) $y = 3x + C$
   (C) $y = 12x + C$
   (D) $y = 3x^2 + C$
   (E) $y = 6x^2 + C$

6. Solve the differential equation $\frac{dy}{dx} = \cos(x)$.

   (A) $y = \cos(x) + C$
   (B) $y = \sin(x) + C$
   (C) $y = -\sin(x) + C$
   (D) $y = -\cos(x) + C$
   (E) $y = x\cos(x) + C$

7. Solve the differential equation $y' = 2x$ given that $y(1) = 6$.

   (A) $y = x^2 + 2$
   (B) $y = x^2 + 3$
   (C) $y = x^2 + 4$
   (D) $y = x^2 + 5$
   (E) $y = 2x^2 + 5$

8. Find the particular solution to $\frac{dy}{dx} = e^x$ satisfying $y(0) = 3$.

   (A) $y = e^x + 3$
   (B) $y = e^x + 2$
   (C) $y = e^x + 1$
   (D) $y = e^x - 3$
   (E) $y = 3e^x$

9. True or False: If $y' = 2y$, then any solution graph must be increasing wherever $y > 0$.

10. True or False: If $y' = -y$, then the solution curves are always decreasing when $y > 0$.

Free Response Practice

1. Verify that $y = e^{3x}$ is a solution to the differential equation:
$$y' = 3y$$

2. Verify that $y = \sin(2x)$ is a solution to the differential equation:
$$y'' + 4y = 0$$

3. Verify that $y = x^2 + 1$ satisfies the differential equation:
$$y' = 2x$$

4. Solve the differential equation $y' = 2x$ given that $y(1) = 5$.

5. Find the particular solution to $\frac{dy}{dx} = 3e^x$ satisfying $y(0) = 2$.

6. Solve $\frac{dy}{dx} = \sin(x)$ with the initial condition $y\left(\frac{\pi}{2}\right) = 0$.