9.1 Differential Equations (Solutions)

Multiple Choice Practice

1. Verify whether $y = e^{2x}$ is a solution to $y' = 2y$.

$$y = e^{2x}, \quad y' = 2e^{2x}$$
$$2y = 2e^{2x} = y'$$

(A) Yes

2. Is $y = x^2 + 1$ a solution to $y' = 2x$?

$$y = x^2 + 1, \quad y' = 2x$$

(A) Yes

3. Which of the following functions satisfy $y' = 3y$? (Select all that apply). **(A) and (B)**

4. Which of the following satisfy $y'' + y = 0$? (Select all that apply). **(A), (B), and (D)**

5. Solve $y' = 6x$.

$$\int dy = \int 6x\, dx$$
$$y = 3x^2 + C$$

(D) $y = 3x^2 + C$

6. Solve $\dfrac{dy}{dx} = \cos(x)$.

$$y = \int \cos(x)\, dx = \sin(x) + C$$

(B) $y = \sin(x) + C$

7. Solve $y' = 2x$, with $y(1) = 6$.

$$\int dy = \int 2x\, dx$$
$$y = x^2 + C$$
$$y(1) = 6 \Rightarrow 1 + C = 6 \Rightarrow C = 5$$
$$\boxed{y = x^2 + 4}$$

(D)

8. Find the particular solution to $\dfrac{dy}{dx} = e^x$, $y(0) = 3$.

$$y = \int e^x\, dx = e^x + C$$
$$y(0) = 3 \Rightarrow 1 + C = 3 \Rightarrow C = 2$$
$$\boxed{y = e^x + 2}$$

(B)

9. True or False: If $y' = 2y$, then any solution graph must be increasing wherever $y > 0$.
True: If $y > 0$, then $y' = 2y > 0$, so the graph increases.

10. True or False: If $y' = -y$, then solution curves are always decreasing when $y > 0$.
True: If $y > 0$, then $y' = -y < 0$, so the graph decreases.

Free Response Practice

1. Verify that $y = e^{3x}$ is a solution to $y' = 3y$.

$$\begin{aligned} y &= e^{3x} \\ y' &= 3e^{3x} = 3y \end{aligned}$$

2. Verify that $y = \sin(2x)$ is a solution to $y'' + 4y = 0$.

$$\begin{aligned} y &= \sin(2x) \\ y' &= 2\cos(2x) \\ y'' &= -4\sin(2x) \\ y'' + 4y &= -4\sin(2x) + 4\sin(2x) = 0 \end{aligned}$$

3. Verify that $y = x^2 + 1$ satisfies $y' = 2x$.

$$\begin{aligned} y &= x^2 + 1 \\ y' &= 2x \end{aligned}$$

4. Solve $y' = 2x$ given $y(1) = 5$.

$$\begin{aligned} \frac{dy}{dx} &= 2x \\ \int dy &= \int 2x\,dx \\ y &= x^2 + C \\ \text{Use } y(1) = 5: \quad 5 &= 1^2 + C \Rightarrow C = 4 \\ \boxed{y &= x^2 + 4} \end{aligned}$$

5. Find the particular solution to $\frac{dy}{dx} = 3e^x$ with $y(0) = 2$.

$$\begin{aligned} \frac{dy}{dx} &= 3e^x \\ \int dy &= \int 3e^x\,dx \\ y &= 3e^x + C \\ \text{Use } y(0) = 2: \quad 2 &= 3e^0 + C = 3 + C \Rightarrow C = -1 \\ \boxed{y &= 3e^x - 1} \end{aligned}$$

6. Solve $\frac{dy}{dx} = \sin(x)$, with $y\left(\frac{\pi}{2}\right) = 0$.

$$\begin{aligned} \frac{dy}{dx} &= \sin(x) \\ \int dy &= \int \sin(x)\,dx \\ y &= -\cos(x) + C \\ \text{Use } y\left(\frac{\pi}{2}\right) = 0: \quad 0 &= -\cos\left(\frac{\pi}{2}\right) + C \Rightarrow C = 0 \\ \boxed{y &= -\cos(x)} \end{aligned}$$