7.4 Partial Fractions

Idea: Partial fractions is for integrals of *rational functions* $\dfrac{P(x)}{Q(x)}$. Rewrite the integrand as a *sum of simpler fractions* and integrate term-by-term.

- If $\deg P \geq \deg Q$: do long division first.
- Factor $Q(x)$ completely.
- Use the templates:

$$\frac{A}{x-a}, \quad \frac{A_1}{x-a} + \cdots + \frac{A_m}{(x-a)^m}, \quad \frac{Ax+B}{x^2+bx+c}, \quad \frac{A_1x+B_1}{x^2+bx+c} + \cdots + \frac{A_mx+B_m}{(x^2+bx+c)^m}.$$

- Solve for constants by clearing denominators, then integrate each term (logs / arctan).

Notes

1. $\displaystyle\int \frac{x^2+2x-1}{2x^3+3x^2-2x}\,dx.$
2. $\displaystyle\int \frac{1}{x^2-9}\,dx.$
3. $\displaystyle\int \frac{x^4+x^3+6x^2+3x+4}{x^3+4x}\,dx.$

WebAssign

1. $\dfrac{x-72}{x^2+x-72}.$
2. $\dfrac{1}{x^2+x^4}.$
3. $\displaystyle\int \frac{37}{(x-1)(x^2+36)}\,dx.$
4. $\dfrac{4y^2-6y-12}{y(y+2)(y-3)}.$
5. $\displaystyle\int \frac{9r^2}{r+2}\,dr.$

Practice

1. $\displaystyle\int \frac{1}{x^2-4}\,dx$
2. $\displaystyle\int \frac{1}{(x-1)(x+2)}\,dx$
3. $\displaystyle\int \frac{1}{x^2+3x+2}\,dx$
4. $\displaystyle\int \frac{2x+3}{x^2+x-2}\,dx$
5. $\displaystyle\int \frac{1}{x^3-x}\,dx$
6. $\displaystyle\int \frac{x^2}{x^3-1}\,dx$
7. $\displaystyle\int \frac{1}{x(x^2+1)}\,dx$
8. $\displaystyle\int \frac{x}{x^3+x^2}\,dx$
9. $\displaystyle\int \frac{x+1}{(x^2-1)(x+3)}\,dx$
10. $\displaystyle\int \frac{x}{(x^2-4)(x+1)}\,dx$