7.3 Trigonometric Substitution

Idea: Trig substitution is for integrals involving $\sqrt{\text{quadratic}}$. Choose a substitution that turns the radical into *one trig function* using a Pythagorean identity.

- $\sqrt{a^2 - x^2}$: let $x = a\sin\theta \quad \Rightarrow \quad \sqrt{a^2 - x^2} = a\cos\theta.$
- $\sqrt{a^2 + x^2}$: let $x = a\tan\theta \quad \Rightarrow \quad \sqrt{a^2 + x^2} = a\sec\theta.$
- $\sqrt{x^2 - a^2}$: let $x = a\sec\theta \quad \Rightarrow \quad \sqrt{x^2 - a^2} = a\tan\theta.$

Rewrite the entire integrand in θ, integrate, then convert back to x (using a right triangle or identities).

Notes

1. $\displaystyle\int \frac{\sqrt{9 - x^2}}{x^2}\,dx.$
2. $\displaystyle\int \frac{1}{x^2\sqrt{x^2 + 4}}\,dx.$
3. $\displaystyle\int \frac{x}{\sqrt{x^2 + 4}}\,dx.$
4. $\displaystyle\int \frac{1}{\sqrt{x^2 - 5}}\,dx.$
5. $\displaystyle\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2 + 9)^{3/2}}\,dx.$

WebAssign

1. $\displaystyle\int \frac{x^2}{\sqrt{x^2 - 3}}\,dx$
2. $\displaystyle\int \frac{x^3}{\sqrt{25 + x^2}}\,dx$
3. $\displaystyle\int_0^7 \frac{dt}{\sqrt{49 + t^2}}$
4. $\displaystyle\int \frac{x}{\sqrt{x^2 - 7}}\,dx$
5. $\displaystyle\int \frac{\sqrt{x^2 - 81}}{x^4}\,dx$

Practice

1. $\displaystyle\int \frac{x}{\sqrt{9 + 3x^2}}\,dx$
2. $\displaystyle\int \frac{x}{(9 + 2x^2)^{3/2}}\,dx$
3. $\displaystyle\int \frac{x^3}{\sqrt{16 + x^2}}\,dx$
4. $\displaystyle\int \sqrt{5 - 2x^2}\,dx$
5. $\displaystyle\int \frac{dx}{\sqrt{5 - 2x^2}}$
6. $\displaystyle\int \frac{dx}{\sqrt{16 + x^2}}$
7. $\displaystyle\int \frac{dx}{\sqrt{8x^2 - 11}}$
8. $\displaystyle\int \frac{1}{x\sqrt{x^2 - 16}}\,dx$
9. $\displaystyle\int \frac{x^2}{\sqrt{10 - 3x^2}}\,dx$
10. $\displaystyle\int \frac{\sqrt{9x^2 - 16}}{x^4}\,dx$
11. $\displaystyle\int \frac{1}{x^2\sqrt{x^2 - 16}}\,dx$
12. $\displaystyle\int_0^{\sqrt{5/2}} \sqrt{5 - 2x^2}\,dx$
13. $\displaystyle\int_0^2 \frac{dx}{\sqrt{16 + x^2}}$
14. $\displaystyle\int \frac{x^2}{(9 + 2x^2)^{3/2}}\,dx$
15. $\displaystyle\int \frac{1}{(25 + 3x^2)^{3/2}}\,dx$