7.2 Trigonometric Integrals

Idea: In trig integrals, your goal is usually to *create a useful derivative factor* and then use identities to rewrite whatever is left.

- $\sin^m x \cos^n x$**:** If *one power is odd*, save one factor of that function and use

  $$\sin^2 x = 1 - \cos^2 x \qquad \text{or} \qquad \cos^2 x = 1 - \sin^2 x$$

  to convert the remaining even power. Then use a u-sub:

  $$\text{save } \cos x \Rightarrow u = \sin x, \qquad \text{save } \sin x \Rightarrow u = \cos x.$$

  If *both powers are even*, use half-angle identities (often repeatedly) to rewrite everything in terms of $\cos(2x)$ and constants.

- $\tan^m x \sec^n x$**:** Try to save a factor of $\sec^2 x$ (so $u = \tan x$), or save a factor of $\sec x \tan x$ (so $u = \sec x$). Then rewrite what remains using

  $$\tan^2 x = \sec^2 x - 1 \qquad \text{or} \qquad \sec^2 x = 1 + \tan^2 x.$$

  In practice: if n is even, save $\sec^2 x$; if m is odd, save $\sec x \tan x$.

Notes

1. $\int \sin^5(x)\cos^2(x)\,dx$
2. $\int \sin^4(x)\,dx$
3. $\int \tan^6(x)\sec^4(x)\,dx$
4. $\int \tan^5(\theta)\sec^7(\theta)\,d\theta$
5. $\int \tan^3(x)\,dx$
6. $\int \sec^3(x)\,dx$

WebAssign

1. $\int 2\sin^2(x)\cos^3(x)\,dx$
2. $\int_0^{\pi/4} \sin^5(x)\,dx$
3. $\int_0^{\pi/2} 3\cos^2(\theta)\,d\theta$
4. $\int_0^{\pi/2} 5\sin^2(x)\cos^2(x)\,dx$
5. $\int 4\tan^3(x)\sec(x)\,dx$
6. $\int 13\tan^4(x)\sec^6(x)\,dx$

Practice

1. $\int \sin^3 x\,dx$
2. $\int \sin^6 x\,dx$
3. $\int \cos^5 x\,dx$
4. $\int \cos^4 x\,dx$
5. $\int \sin^2 x\,\cos x\,dx$
6. $\int \sin x\,\cos^2 x\,dx$
7. $\int \sin^4 x\,\cos^3 x\,dx$
8. $\int \sin^3 x\,\cos^4 x\,dx$
9. $\int \sin^2 x\,\cos^2 x\,dx$
10. $\int \tan^2 x\,dx$
11. $\int \sec^4 x\,dx$
12. $\int \tan^4 x\,\sec^2 x\,dx$
13. $\int \tan^3 x\,\sec^3 x\,dx$
14. $\int \tan^2 x\,\sec^3 x\,dx$