7.1 Integration by Parts

Every differentiation rule has a corresponding integration rule. For instance, the Substitution Rule for integration corresponds to the Chain Rule for differentiation. The rule that corresponds to the Product Rule for differentiation is called *integration by parts*.

Theorem (Integration by Parts, Formula 1)**.** If f and g are differentiable functions, then

$$\int f(x)\, g'(x)\, dx \;=\; f(x)\, g(x) \;-\; \int f'(x)\, g(x)\, dx.$$

The formula for integration by parts is often written in the following form:

Theorem (Integration by Parts, Formula 2)**.** Let $u = f(x)$ and $v = g(x)$ in the theorem above. Then $du = f'(x)\, dx$ and $dv = g'(x)\, dx$. We obtain:

$$\int u\, dv \;=\; u\, v \;-\; \int v\, du.$$

Example. Find $\int x \sin x \, dx$.

Question. In the above example, what would have happened if we had instead chosen $u = \sin x$ and $dv = x \, dx$?

Example. Evaluate $\int \ln x \, dx$.

Example. Find $\int t^2 e^t \, dt$.

Example. Evaluate $\int e^x \sin x \, dx$.

Definite Integration by Parts

If we combine the formula for integration by parts with the Evaluation Theorem, we can evaluate definite integrals by parts:

Theorem (Definite Integration by Parts)**.**

$$\int_a^b f(x)\, g'(x)\, dx \;=\; \Big[f(x)\, g(x)\Big]_a^b \;-\; \int_a^b f'(x)\, g(x)\, dx.$$

Example. Calculate $\displaystyle\int_0^1 \tan^{-1}(x)\, dx$.