

## 7.1 Integration by Parts

Integration by Parts is the *product rule in reverse*. Use it when an integrand is a *product* of two functions and differentiating one factor makes the problem simpler.

**Indefinite formula.**

$$\int u dv = uv - \int v du.$$

(For indefinite integrals, we usually add a single  $+C$  at the end of the final antiderivative.)

**Definite formula.** If the integral has bounds  $[a, b]$ , then

$$\int_a^b u dv = [uv]_a^b - \int_a^b v du, \quad \text{where} \quad [uv]_a^b = u(b)v(b) - u(a)v(a).$$

Choose  $u$  so that  $du$  is simpler than  $u$ , and choose  $dv$  so that you can compute  $v = \int dv$  easily. The goal is that the new integral  $\int v du$  is simpler than the original. A common heuristic for choosing  $u$  is **LIATE**, listed from *highest* to *lowest* priority:

$$\boxed{\mathbf{L} > \mathbf{I} > \mathbf{A} > \mathbf{T} > \mathbf{E}}$$

- **L**ogarithmic:  $\ln x$
- **I**nverse trig:  $\arctan x$ ,  $\arcsin x$ , etc.
- **A**lgebraic: powers of  $x$ , polynomials, roots
- **T**rig:  $\sin x$ ,  $\cos x$ , etc.
- **E**xponential:  $e^x$ ,  $a^x$

### Notes

1.  $\int x \sin x dx$ .
2.  $\int x^2 \ln x dx$ .
3.  $\int \ln x dx$ .
4.  $\int t^2 e^t dt$ .
5.  $\int_0^1 \tan^{-1}(x) dx$ .

### WebAssign

1.  $\int x e^{7x} dx$ .
2.  $\int \arctan(5t) dt$ .
3.  $\int 6 \arcsin x dx$ .
4.  $\int (x^2 + 4x) \cos(x) dx$ .

### Practice

1.  $\int x e^x dx$
2.  $\int x \ln(x) dx$
3.  $\int x^2 \sin(x) dx$
4.  $\int x \cos(x) dx$
5.  $\int x^3 \ln(x) dx$
6.  $\int x^2 \ln(x^2) dx$
7.  $\int x^2 \cos(2x) dx$
8.  $\int \ln(x^2 + 1) dx$

## 7.1 Boomerang Integrals

Sometimes Integration by Parts produces an integral that is essentially the same as the one you started with. When that happens, the integral “boomerangs” back. The key is to recognize the repeat, apply IBP enough times to make the original integral reappear, and then solve algebraically.

Boomerang integrals often occur with products involving trig and exponential functions, such as

$$\int e^{ax} \sin(bx) dx, \quad \int e^{ax} \cos(bx) dx,$$

because two rounds of IBP typically bring you back to a constant multiple of the original integral. They can also appear in repeated IBP patterns like  $\int (\ln x)^n dx$ .

### Notes

1.  $\int e^x \sin(x) dx$ .
2.  $\int \sec^3(x) dx$ .<sup>†</sup>

### Practice

1.  $\int e^{3x} \cos(3x) dx$
2.  $\int e^{-x} \sin(x) dx$

### WebAssign

1.  $\int e^{2\theta} \sin(3\theta) d\theta$

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<sup>†</sup>We did this in Section 7.2.