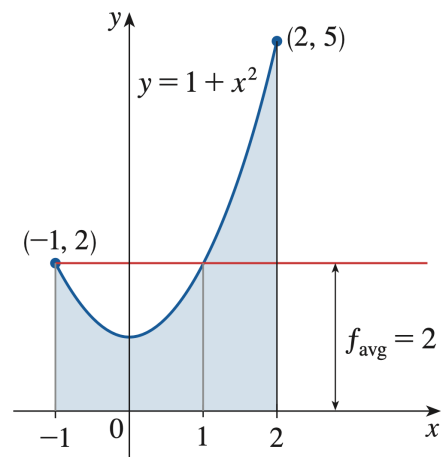


## 6.5 Average Value of a Function

**Theorem.** The average value of  $f$  on the interval  $[a, b]$  is  $f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx$ .

- The average of  $n$  numbers  $x_1, x_2, \dots, x_n$  is given by:
- For functions, the idea of “average” extends to infinitely many values (since a function takes on a value at every point in the interval  $[a, b]$ ). We use an integral to represent the total sum.
- The average value requires dividing the total sum by the “number of values.” For functions over  $[a, b]$ , the equivalent is dividing by the length of the interval  $b - a$ .

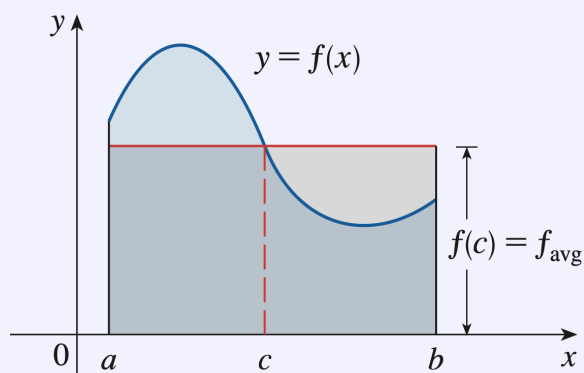
**Example.** Find the average value of the function  $f(x) = 1 + x^2$  on the interval  $[-1, 2]$ .



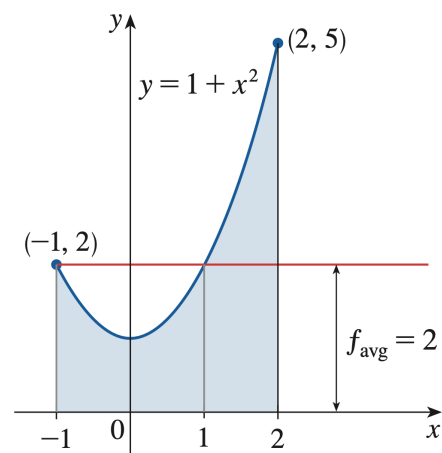
**Theorem** (The Mean Value Theorem for Integrals). If  $f$  is continuous on  $[a, b]$ , then there exists a number  $c$  in  $[a, b]$  such that

$$f(c) = f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) \, dx$$

that is,  $\int_a^b f(x) \, dx = f(c)(b-a)$



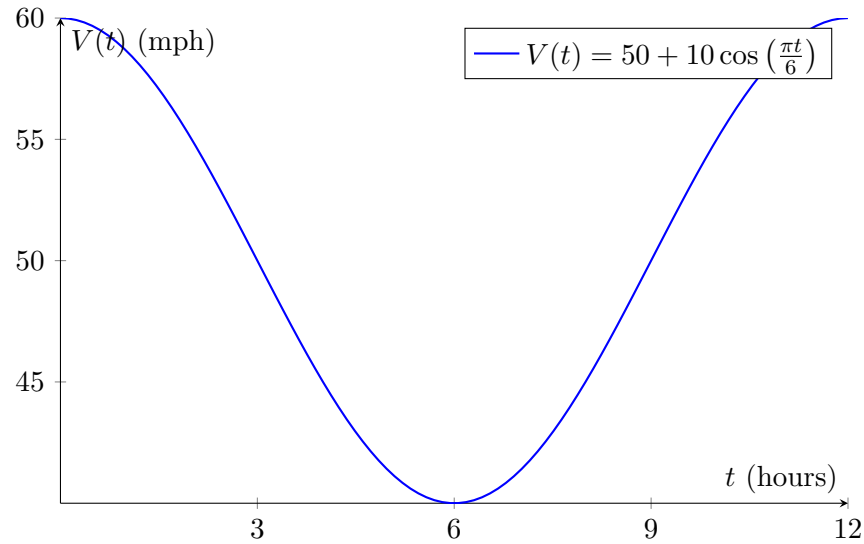
**Example.** Determine the value of  $c$  that satisfies the conclusion of the Mean Value Theorem for the function  $f(x) = 1 + x^2$  on the interval  $[-1, 2]$ .



**Example.** Show that the average velocity of a car over a time interval  $[t_1, t_2]$  is the same as the average of its velocities during the trip.

**Example** (Monitoring Traffic Flow). Traffic engineers are analyzing vehicle speeds on a 6-mile stretch of highway during a 12-hour observation period. The speed of traffic (in miles per hour) at time  $t$  (in hours) is modeled as:

$$V(t) = 50 + 10 \cos\left(\frac{\pi t}{6}\right), \quad t \in [0, 12].$$



**Question.** Knowing the average speed helps ensure that traffic flows smoothly and within safe limits, preventing dangerous speed fluctuations. What is the average speed of vehicles over the 12-hour period?

**Question.** At what times does the instantaneous speed match the average speed? These moments indicate when traffic flow is most representative of overall conditions, helping engineers optimize traffic signals and safety measures.