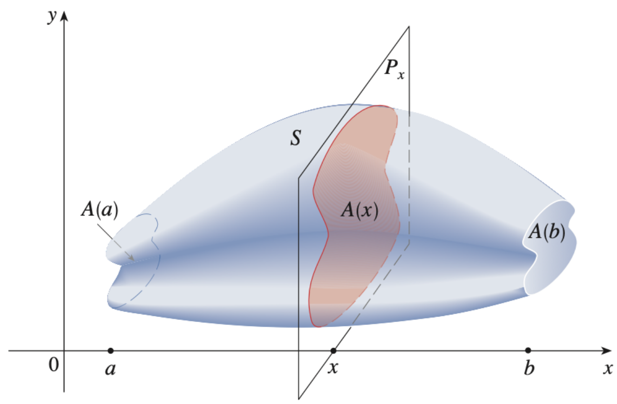
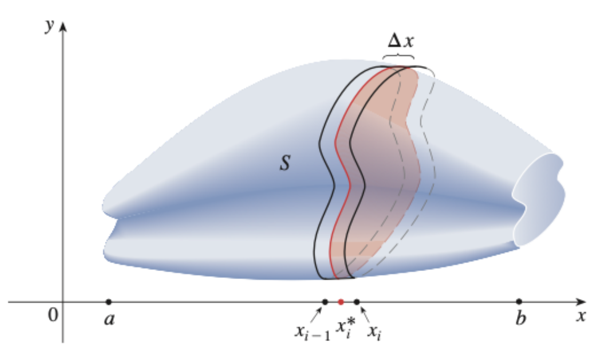
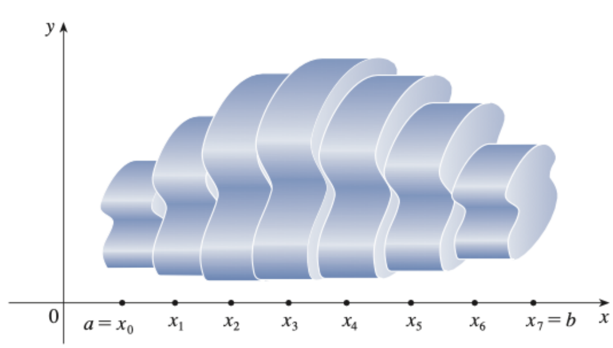
6.2 Volumes

Definition of Volume

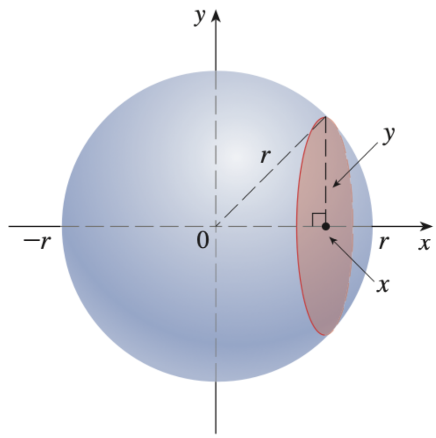
Question. How can we find the volume of a solid region S?

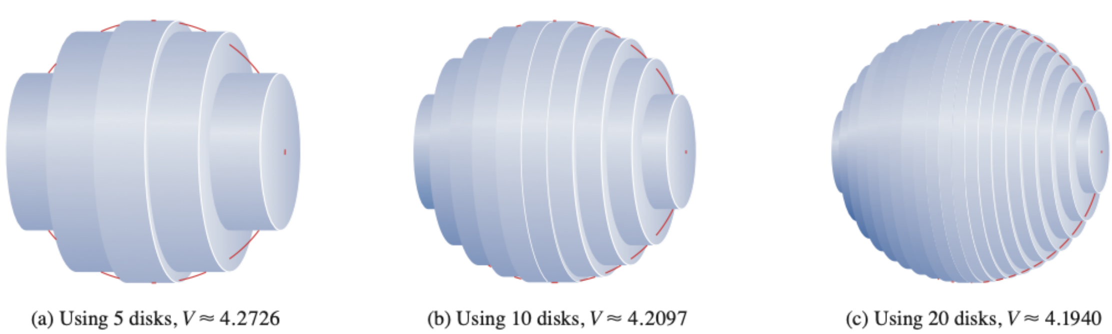






Example. Show that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

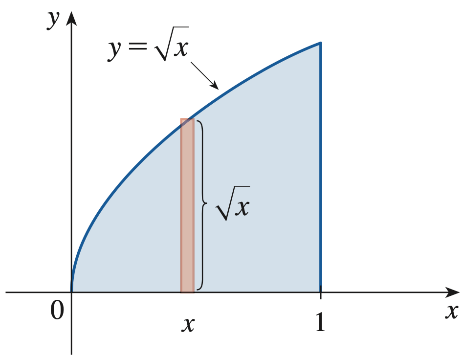


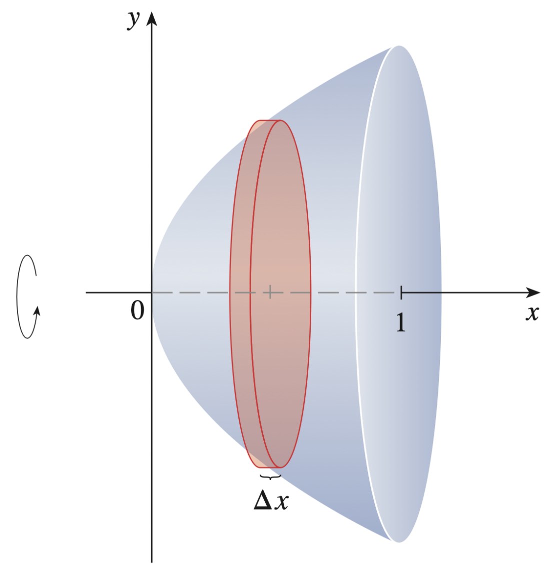


Approximating the volume of a sphere with radius 1

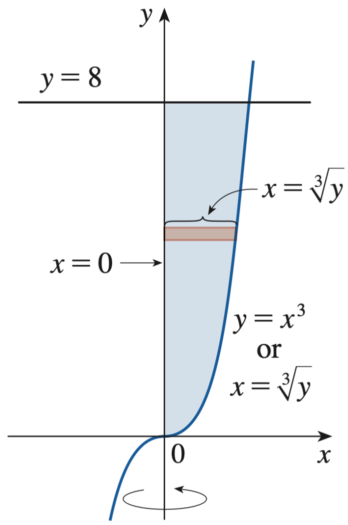
Volumes of Solids of Revolution

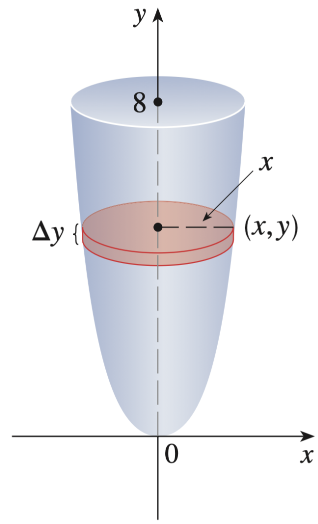
Example. Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.



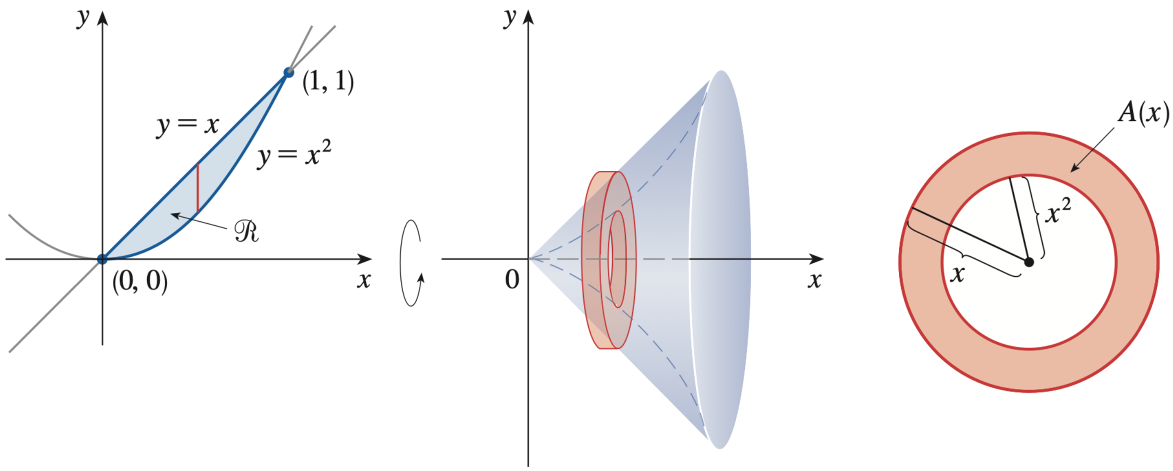


Example. Find the volume of the solid obtained by rotating the region bounded by $y = x^3, y = 8$, and $x = 0$ about the y-axis.

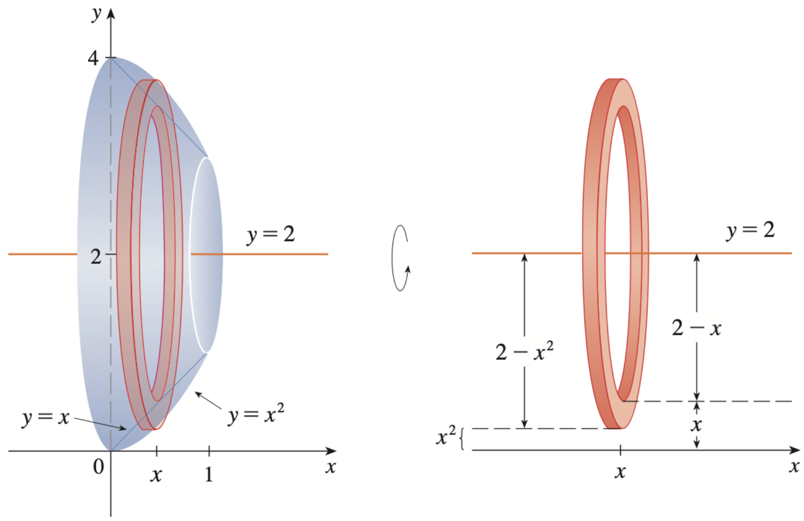




Example. The region R enclosed by the curves $y = x$ and $y = x^2$ is rotated about the x-axis. Find the volume of the resulting solid.



Example. Find the volume of the solid obtained by rotating the region R enclosed by the curves $y = x$ and $y = x^2$ about the line $y = 2$.



Summary

To calculate the volume of a solid of revolution, we use the defining formulas:

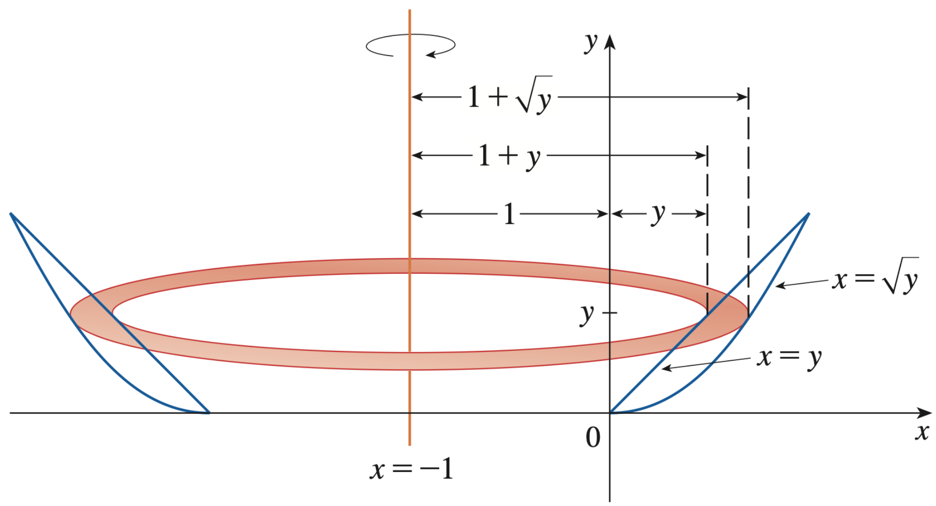
$$V = \int_a^b A(x)\,dx \quad \text{(rotation around the x-axis)}$$

$$V = \int_c^d A(y)\,dy \quad \text{(rotation around the y-axis)}$$

$A(x)$ or $A(y)$ represents the cross-sectional area, which is determined by the method used.

Cross-Section	Process for Finding Area
Disk	Cross-section is a solid disk. Determine the radius $r(x)$ or $r(y)$ based on the axis of rotation. Compute the area of the disk using: $A = \pi \cdot [\text{radius}]^2$
Washer	Cross-section is a washer (a disk with a hole). Find the inner radius r_{in} and outer radius r_{out} from a sketch or equation. Compute the area of the washer by subtracting the inner disk area from the outer disk area: $A = \pi \cdot [r_{\text{out}}]^2 - \pi \cdot [r_{\text{in}}]^2$

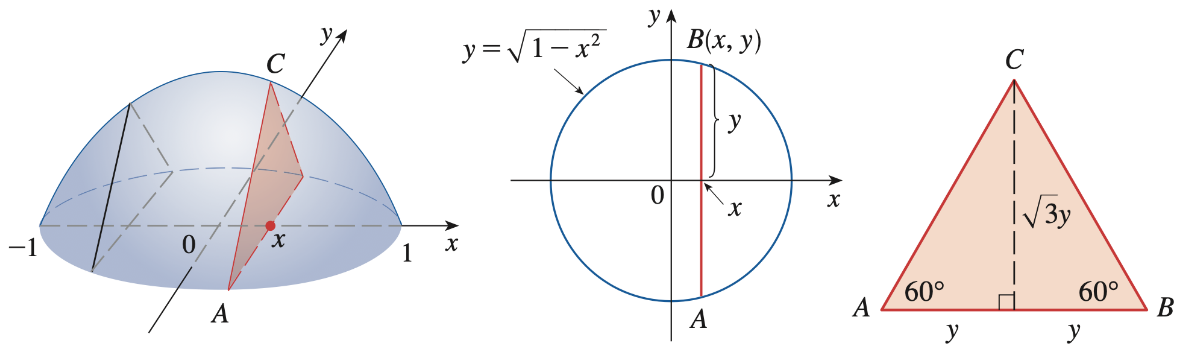
Example. Find the volume of the solid obtained by rotating the region R enclosed by the curves $y = x$ and $y = x^2$ about the line $x = -1$.



Finding Volume Using Cross-Sectional Area

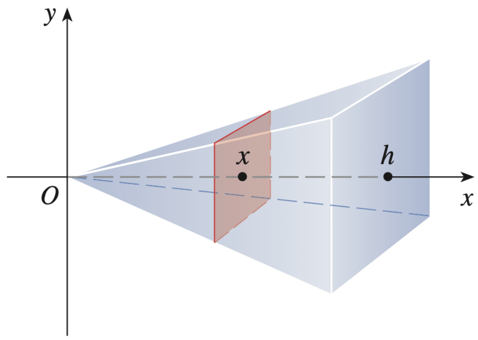
We now find the volumes of solids that are not solids of revolution but whose cross-sections have areas that are readily computable.

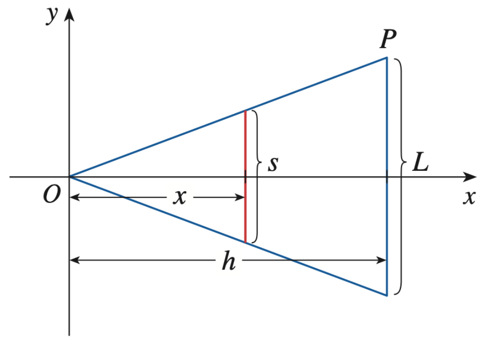
Example. The figure below shows a solid with a circular base of radius 1. Parallel cross-sections perpendicular to the base are equilateral triangles. Find the volume of the solid.



(a) The solid (b) Its base (c) A cross-section

Example. Find the volume of a pyramid whose base is square with side L and whose height is h.





Example. A wedge is cut out of a circular cylinder of radius 4 by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of 30° along a diameter of the cylinder. Find the volume of the wedge.