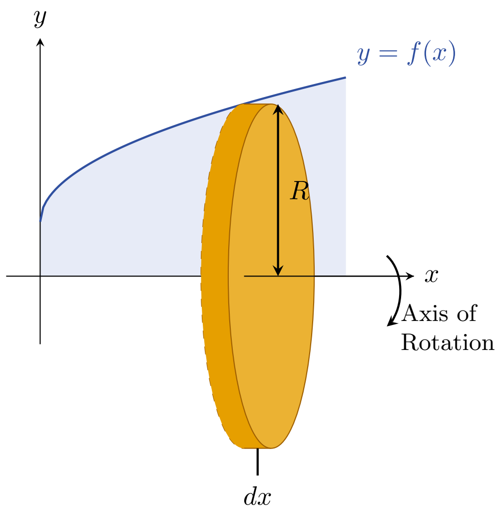
## 6.2 Volumes of Solids of Revolution

When a region is revolved around an axis, we can compute the volume by adding up thin circular cross-sections.

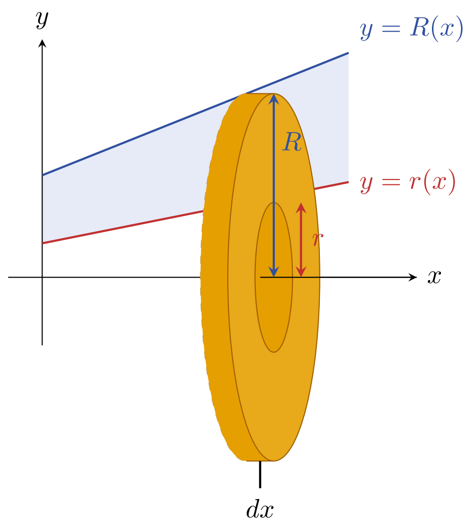
**1. Disk method:** cross-sections are *solid* circles.

$$V = \pi \int_a^b \big(R(x)\big)^2\,dx \qquad \text{or} \qquad V = \pi \int_c^d \big(R(y)\big)^2\,dy$$

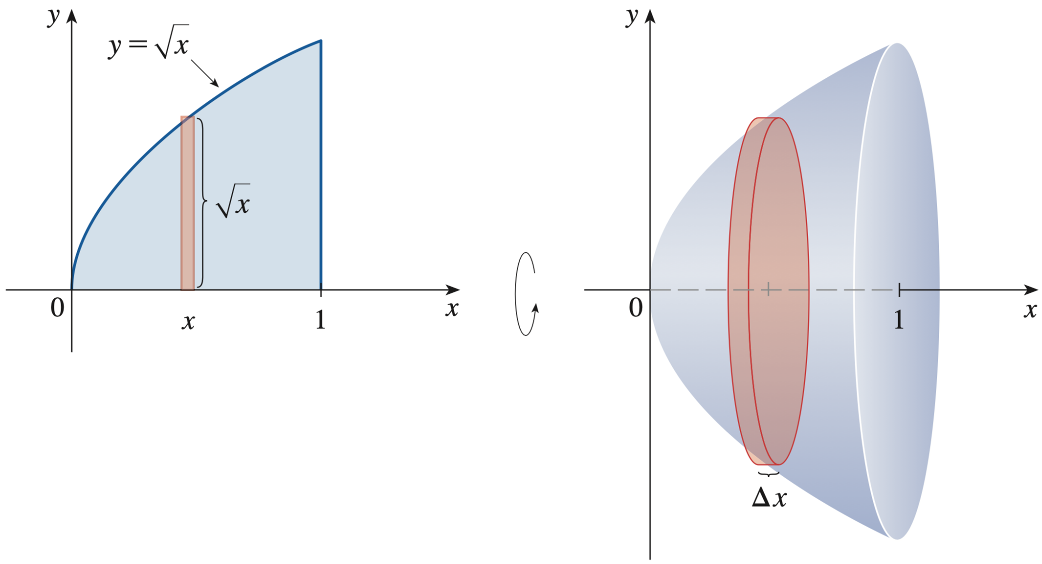



**2. Washer method:** cross-sections have a *hole*.

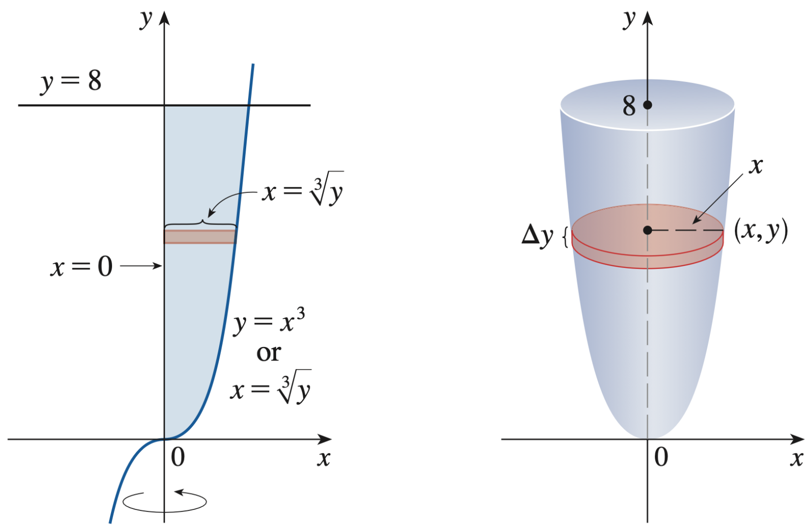
$$V = \pi \int_a^b \Big(\big(R(x)\big)^2 - \big(r(x)\big)^2\Big)\,dx \qquad \text{or} \qquad V = \pi \int_c^d \Big(\big(R(y)\big)^2 - \big(r(y)\big)^2\Big)\,dy$$



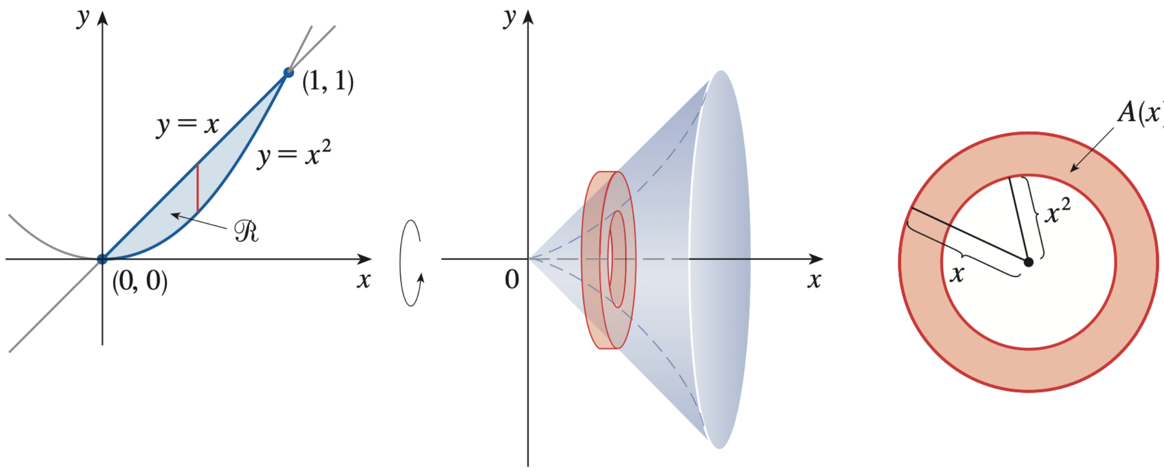
**Example.** Find the volume of the solid obtained by rotating about the $x$-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.



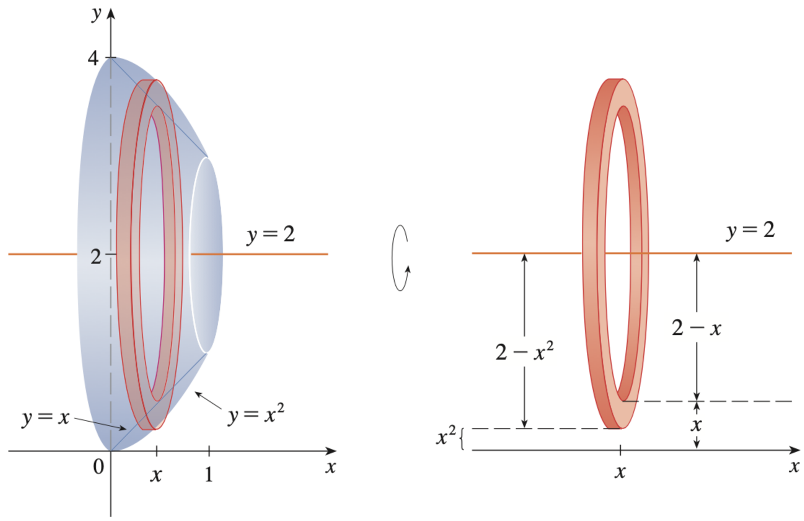
**Example.** Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the $y$-axis.



**Example.** The region $R$ enclosed by the curves $y = x$ and $y = x^2$ is rotated about the $x$-axis. Find the volume of the resulting solid.



**Example.** Find the volume of the solid obtained by rotating the region $R$ enclosed by the curves $y = x$ and $y = x^2$ about the line $y = 2$.



**Example.** Find the volume of the solid obtained by rotating the region $R$ enclosed by the curves $y = x$ and $y = x^2$ about the line $x = -1$.