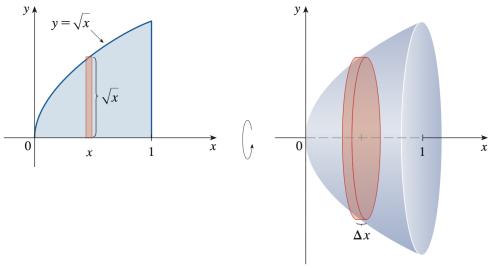
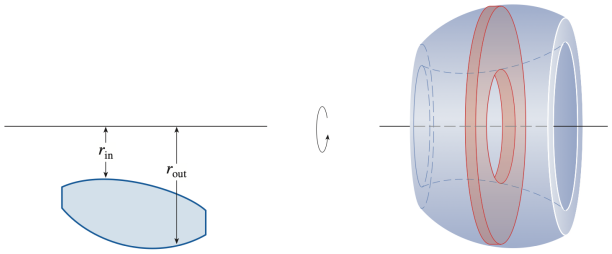
6.2-6.3 Solids of Revolution

There are three primary slicing methods used to compute the volume of a solid of revolution:

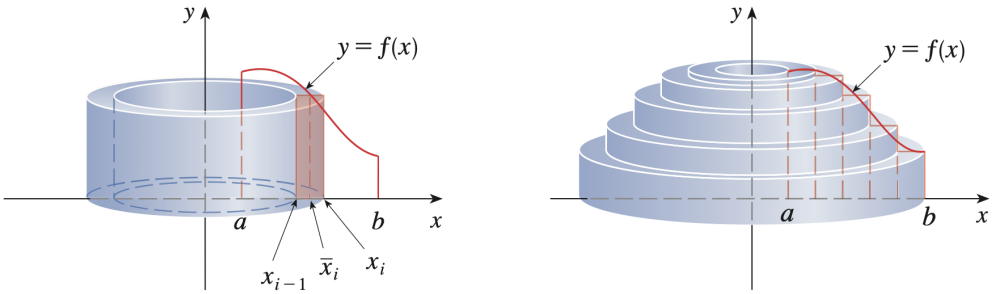
Disk Method: Slices are drawn perpendicular to the axis of rotation. If there is no gap between the region and the axis, each slice is disk.



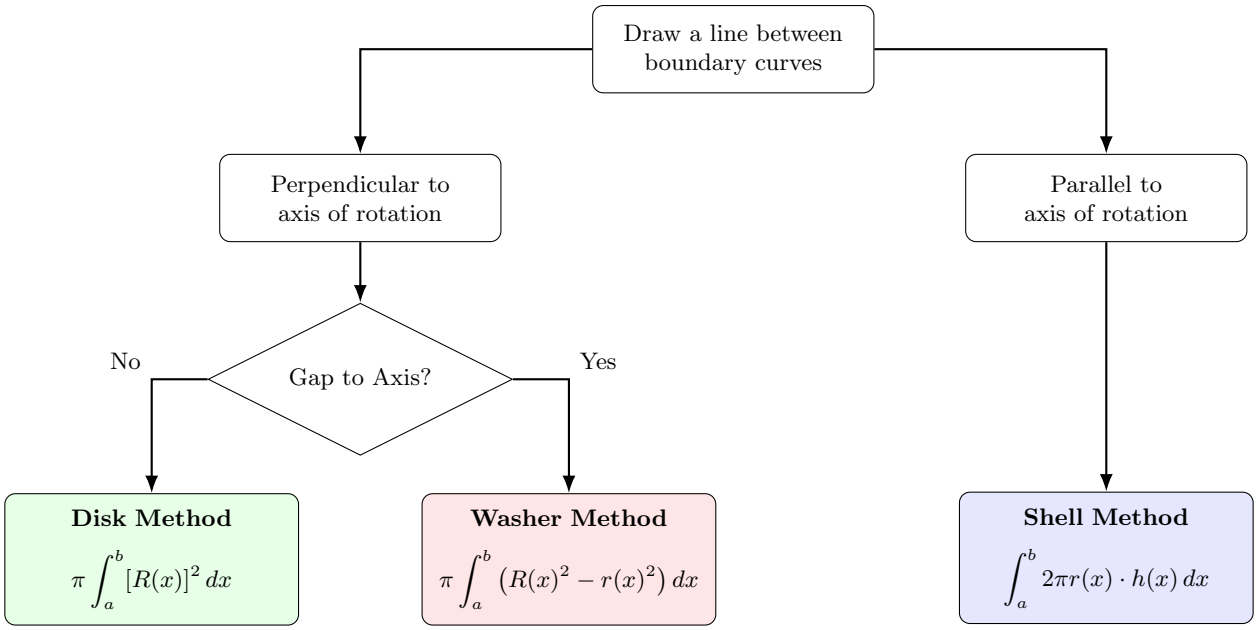
Washer Method: Slices are drawn perpendicular to the axis of rotation. If there is a gap between the region and the axis, each slice is washer.



Shell Method: Slices are drawn parallel to the axis of rotation. Each slice forms a cylindrical shell, and its volume is determined by its radius, height, and thickness.



This flowchart helps you choose the correct setup for a volume of revolution problem.



Volume Problems

Disk Method

1. Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x}, \quad y = 0, \quad x = 4$$

about the x-axis.

2. Find the volume of the solid obtained by rotating the region bounded by

$$y = 2 - x^2, \quad y = 0$$

about the x-axis.

3. Find the volume of the solid obtained by rotating the region bounded by

$$x = y^2, \quad x = 0, \quad y = 3$$

about the y-axis.

4. Find the volume of the solid obtained by rotating the region bounded by

$$y = 2x, \quad x = 0, \quad y = 4$$

about the y-axis.

5. Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x - 3}, \quad y = 0, \quad x = 7$$

about the vertical line $x = 7$.

6. Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x + 1}, \quad y = 0, \quad x = 3$$

about the vertical line $x = -1$.

7. Find the volume of the solid obtained by rotating the region bounded by

$$y = 2 - \sqrt{x}, \quad y = 2, \quad x = 1$$

about the horizontal line $y = 2$.

8. Find the volume of the solid obtained by rotating the region bounded by

$$y = -1 + \sqrt{x}, \quad y = -1, \quad x = 1$$

about the horizontal line $y = -1$.

Washer Method

1. Find the volume of the solid obtained by rotating the region bounded by

$$y = x, \quad y = x^2$$

about the x-axis.

2. Find the volume of the solid obtained by rotating the region bounded by

$$y = 2 - x, \quad y = x, \quad x = 0$$

about the x-axis.

3. Find the volume of the solid obtained by rotating the region bounded by

$$x = y, \quad x = y^2$$

about the y-axis.

4. Find the volume of the solid obtained by rotating the region bounded by

$$y = x, \quad y = 4 - x, \quad \text{and} \quad y = 0$$

about the y-axis.

5. Find the volume of the solid obtained by rotating the region bounded by

$$x = y^2, \quad x = 5 - y^2, \quad y = -1, \quad \text{and} \quad y = 1$$

about the vertical line $x = 7$.

6. Find the volume of the solid obtained by rotating the region bounded by

$$y = x - 1, \quad y = 4 - x, \quad y = 0, \quad \text{and} \quad y = 1$$

about the vertical line $x = -1$.

7. Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2, \quad y = \sqrt{x}$$

about the horizontal line $y = 2$.

8. Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x}, \quad y = \frac{x}{2}$$

about the horizontal line $y = -1$.

Cylindrical Shell Method

1. Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x}, \quad y = 0, \quad x = 4$$

about the x-axis.

2. Find the volume of the solid obtained by rotating the region bounded by

$$y = 2 - x, \quad y = 0, \quad x = 0$$

about the x-axis.

3. Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2, \quad y = 4, \quad x = 0$$

about the y-axis.

4. Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x}, \quad y = 0, \quad x = 9$$

about the y-axis.

5. Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x}, \quad y = 0, \quad x = 1, \quad x = 4$$

about the vertical line $x = -1$.

6. Find the volume of the solid obtained by rotating the region bounded by

$$y = x + 2, \quad y = 0, \quad x = -1, \quad x = 3$$

about the vertical line $x = 5$.

7. Find the volume of the solid obtained by rotating the region bounded by

$$x = y^2, \quad x = 4, \quad y = 0$$

about the horizontal line $y = 3$.

8. Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x}, \quad y = \frac{x}{2}$$

about the horizontal line $y = -2$.

Additional Problems

1. Find the volume of the solid obtained by rotating the region bounded by

$$x = y^2, \quad x = 4$$

about the y-axis.

2. Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2, \quad y = 2 - x$$

about the vertical line $x = -2$.

3. Find the volume of the solid obtained by rotating the region bounded by

$$y = \sin x, \quad y = 0, \quad x = 0, \quad x = \pi$$

about the line $y = 1$.

4. Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2, \quad y = \sqrt{x}$$

about the y-axis.

5. Find the volume of the solid obtained by rotating the region bounded by

$$x = 1 - y^2, \quad x = 0$$

about the y-axis.

6. Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2, \quad y = 2 - x^2$$

about the line $y = -1$.

7. Find the volume of the solid obtained by rotating the region bounded by

$$y = \ln x, \quad y = 0, \quad x = 1, \quad x = e^2$$

about the y-axis.

8. Find the volume of the solid obtained by rotating the region bounded by

$$y = \frac{4}{x}, \quad y = 0, \quad x = 1, \quad x = 4$$

about the line $x = -2$.