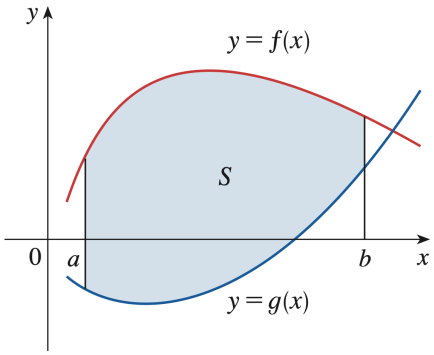
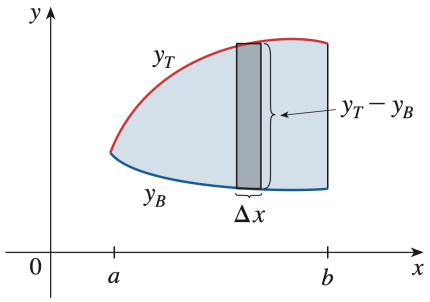
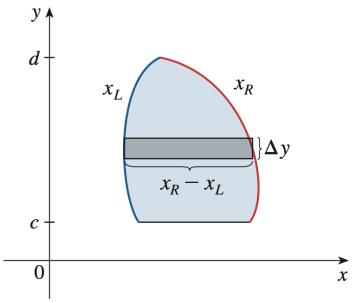
6.1 Areas Between Curves

1. Visualize the Problem

- Sketch the curves to understand their relative positions.
- Identify the region whose area needs to be calculated.



2. Decide the Variable of Integration

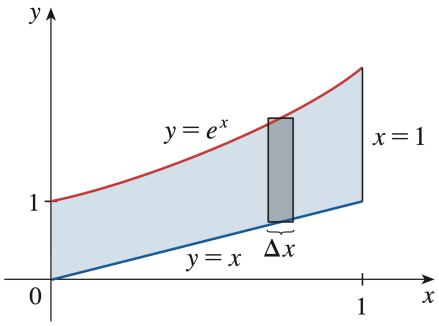
Integration with Respect to x	Integration with Respect to y
Choose to integrate with respect to x if the region is vertically bounded (i.e., top and bottom curves).	Choose to integrate with respect to y if the region is horizontally bounded (i.e., right and left curves).
	
The area is given by: $$\text{Area} = \int_a^b (y_T - y_B)\, dx$$	The area is given by: $$\text{Area} = \int_c^d (x_R - x_L)\, dy$$

3. Find the Limits of Integration

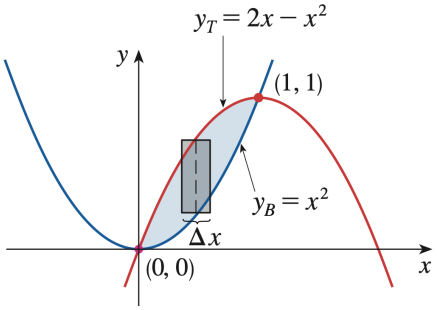
- These are either given, or they correspond to points of intersection.
- If needed, solve for the intersection points of the curves by setting them equal.

Area Between Curves: Integrating With Respect to x

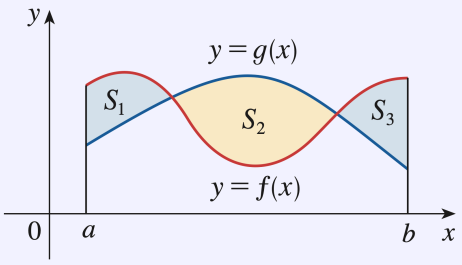
Example. Find the area of the region bounded above by $y = e^x$, bounded below by $y = x$, and bounded on the sides by $x = 0$ and $x = 1$.



Example. Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.



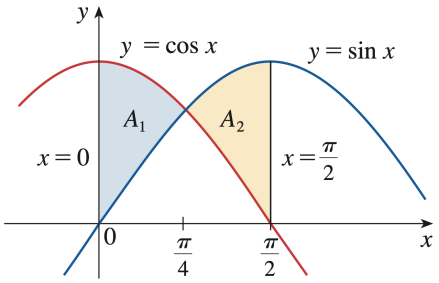
Sometimes we are asked to find the area between the curves $y = f(x)$ and and $y = g(x)$ where $f(x) \geq g(x)$ for some values of x but $g(x) \geq f(x)$ for other values of x.



In this case, we split the given region S into several regions $S_1, S_2, \ldots$ with areas $A_1, A_2, \ldots$. The total area is

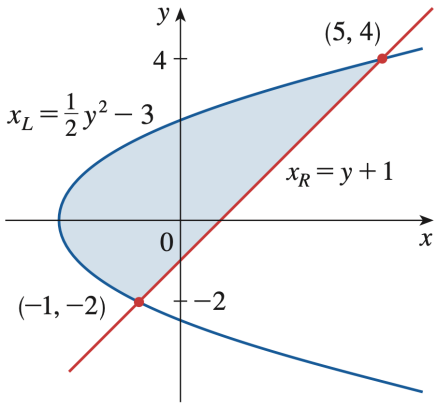
$$A = A_1 + A_2 + \cdots = \int_a^b |f(x) - g(x)|\, dx$$

Example. Find the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \pi/2$.



Area Between Curves: Integrating With Respect to y

Example. Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.



Example. Find the area of the region enclosed by the curves $y = 1/x$, $y = x$, and $y = \frac{1}{4}x$, using

(a) x as the variable of integration.

(b) y as the variable of integration.