6.1 Areas Between Curves

Area is always *(bigger boundary)* − *(smaller boundary)* integrated over the correct interval. You can slice the region two ways:

- **Vertical slices (dx):** write the boundaries as y-functions of x. If the region runs from $x = a$ to $x = b$, then

$$A = \int_a^b \left[y_T(x) - y_B(x)\right] dx \qquad \text{(top minus bottom)}.$$

- **Horizontal slices (dy):** write the boundaries as x-functions of y. If the region runs from $y = c$ to $y = d$, then

$$A = \int_c^d \left[x_R(y) - x_L(y)\right] dy \qquad \text{(right minus left)}.$$

Important: if the curves cross inside the interval, the "top/bottom" (or "right/left") ordering changes. Find the intersection value(s) and *split the integral* so the ordering stays consistent on each piece (or use absolute value with the same split points).

Practice

1. Find the area enclosed by the curves $y = x^2$ and $y = 4$.

2. Find the area between the curves $y = x^3$ and $y = x$.

3. Compute the area enclosed by the curves $y = \sin x$ and $y = \cos x$ over the interval $0 \le x \le \frac{\pi}{2}$.

4. Determine the area of the region bounded by $y = x^2 + 1$ and $y = 3$.

5. Find the area enclosed by the curves $y = e^x$ and $y = 4$ for $0 \le x \le \ln 4$.