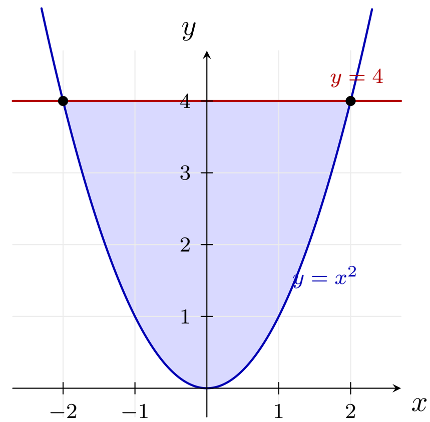
6.1 Areas Between Curves (Solutions)

1. **Find the area enclosed by the curves $y = x^2$ and $y = 4$.**

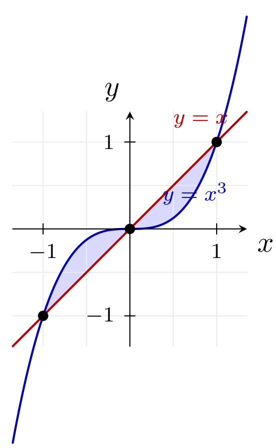


Solve $x^2 = 4$ to get $x = \pm 2$. On $[-2, 2]$, the top curve is $y = 4$ and the bottom curve is $y = x^2$, so

$$A = \int_{-2}^{2} (4 - x^2)\,dx = 2\int_{0}^{2} (4 - x^2)\,dx = 2\left[4x - \frac{x^3}{3}\right]_0^2 = \frac{32}{3}.$$

$$\boxed{A = \frac{32}{3}}$$

2. **Find the area between the curves $y = x^3$ and $y = x$.**

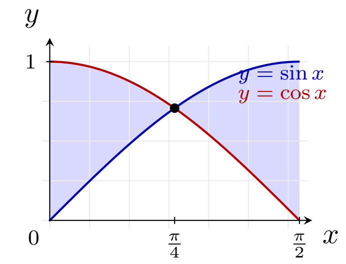


Solve $x^3 = x$: $x(x^2 - 1) = 0$, so $x = -1, 0, 1$. On $[0, 1]$, $x \geq x^3$, and symmetry gives

$$A = 2\int_{0}^{1} (x - x^3)\,dx = 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2}.$$

$$\boxed{A = \frac{1}{2}}$$

3. **Compute the area enclosed by $y = \sin x$ and $y = \cos x$ on $0 \le x \le \frac{\pi}{2}$.**

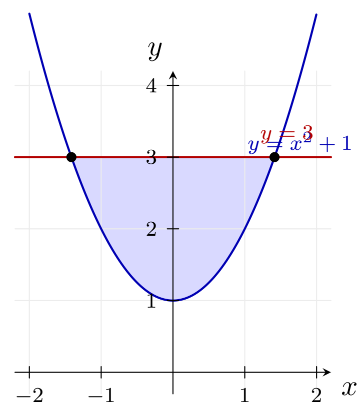


The curves meet when $\sin x = \cos x$, which occurs at $x = \frac{\pi}{4}$. On $\left[0, \frac{\pi}{4}\right]$, $\cos x \ge \sin x$, and the two pieces have equal area, so

$$A = 2\int_0^{\pi/4} \big(\cos x - \sin x\big)\, dx = 2\left[\sin x + \cos x\right]_0^{\pi/4} = 2(\sqrt{2} - 1).$$

$$\boxed{A = 2(\sqrt{2} - 1)}$$

4. **Determine the area of the region bounded by $y = x^2 + 1$ and $y = 3$.**



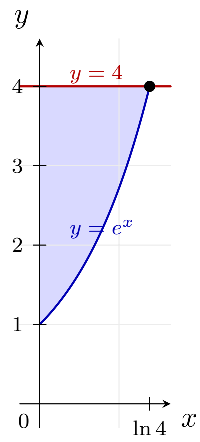
Intersect where $x^2 + 1 = 3$, so $x = \pm\sqrt{2}$. The top curve is $y = 3$ and the bottom curve is $y = x^2 + 1$, hence

$$A = \int_{-\sqrt{2}}^{\sqrt{2}} \Big(3 - (x^2 + 1)\Big)\, dx = \int_{-\sqrt{2}}^{\sqrt{2}} (2 - x^2)\, dx = 2\int_0^{\sqrt{2}} (2 - x^2)\, dx.$$

$$A = 2\left[2x - \frac{x^3}{3}\right]_0^{\sqrt{2}} = \frac{8\sqrt{2}}{3}.$$

$$\boxed{A = \frac{8\sqrt{2}}{3}}$$

5. **Find the area enclosed by $y = e^x$ and $y = 4$ for $0 \le x \le \ln 4$.**



On $[0, \ln 4]$, the top curve is $y = 4$ and the bottom curve is $y = e^x$, so

$$A = \int_0^{\ln 4} \left(4 - e^x\right) dx = \left[4x - e^x\right]_0^{\ln 4} = 4\ln 4 - 3.$$

$$\boxed{A = 4\ln 4 - 3}$$