5.5 u-Substitution

This goal is to rewrite an integral in a simpler variable by spotting an "inside function" whose derivative (or a constant multiple of it) also appears in the integrand. General procedure:

1. **Choose u:** pick a function built from the complicated part of the integrand—often what is *inside* a power, root, exponential, or logarithm.
2. **Differentiate:** compute $du = u'(x)\,dx$.
3. **Match du:** rewrite the integral so that a factor of $u'(x)\,dx$ is present, then replace it by du.
4. **Substitute:** replace every remaining x-expression with u, producing an integral in u.
5. **Integrate and back-substitute:** integrate with respect to u, then substitute $u = g(x)$ back in.
6. **If bounds are given:** either convert the bounds to u-values (recommended), or back-substitute first and then evaluate in x.

Notes

1. $\displaystyle\int x^3 \cos(x^4 + 2)\ dx$
2. $\displaystyle\int \frac{x}{\sqrt{1 - 4x^2}}\ dx$
3. $\displaystyle\int \tan(x)\,dx$
4. $\displaystyle\int e^{5x}\ dx$
5. $\displaystyle\int \frac{e^{1/x}}{x^2}\,dx$
6. $\displaystyle\int x\sqrt{x-1}\,dx$
7. $\displaystyle\int x^5\sqrt{1+x^2}\ dx$
8. $\displaystyle\int_0^4 \sqrt{2x+1}\ dx$
9. $\displaystyle\int_0^2 xe^{x^2}\,dx$
10. $\displaystyle\int_1^e \frac{\ln x}{x}\ dx$

WebAssign

1. $\displaystyle\int \frac{x^3}{x^4 - 4}\,dx$
2. $\displaystyle\int \frac{\cos(\sqrt{t})}{\sqrt{t}}\,dt$
3. $\displaystyle\int (3 - 8x)^{10}\,dx$
4. $\displaystyle\int \frac{(\ln(x))^{30}}{x}\,dx$
5. $\displaystyle\int y^2\left(5 - y^3\right)^{2/3}\,dy$
6. $\displaystyle\int_0^1 \sqrt[3]{1+7x}\,dx$
7. $\displaystyle\int_1^6 \frac{e^{1/x}}{x^2}\,dx$
8. $\displaystyle\int_0^{\pi/6} \frac{\sin(t)}{\cos^2(t)}\,dt$

Practice

1. $\displaystyle\int x\sqrt{1 - x^2}\,dx$
2. $\displaystyle\int x\sec^2(x^2)\,dx$
3. $\displaystyle\int \frac{e^x}{1 + e^x}\,dx$
4. $\displaystyle\int \frac{x}{(x^2+1)^3}\,dx$
5. $\displaystyle\int \frac{x}{1 + x^4}\,dx$
6. $\displaystyle\int \frac{x}{\sqrt{x+2}}\,dx$
7. $\displaystyle\int_0^1 (5x+1)^3\,dx$
8. $\displaystyle\int_0^{\pi/4} \sec^2(x)\tan(x)\,dx$
9. $\displaystyle\int_0^{\sqrt{7}} x\sqrt{x^2+1}\,dx$
10. $\displaystyle\int_0^{\ln 2} e^{-x}\,dx$