11.8 Power Series

A **power series** is a series of the form

$$\sum_{n=0}^{\infty} c_n(x-a)^n,$$

where a is the **center** and the c_n's are constants.

The **radius of convergence**, denoted R, tells how far the series converges away from its center:

$$|x-a| < R \;\Rightarrow\; \text{converges}, \qquad |x-a| > R \;\Rightarrow\; \text{diverges}.$$

If $|x-a| = R$, you must test the endpoint(s) separately.

The **interval of convergence** is the set of all x-values for which the series converges. It is built from the open interval

$$(a-R, a+R)$$

and then adjusted after checking the endpoints.

Practice Problems

For each series below, find the radius of convergence R and the interval of convergence I.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(2x-5)^n}{n \cdot 3^n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2(x+4)^n}{7^n}$
3. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^{2n}}{5^n}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(3x)^n}{n^2}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{(n+2)!}$
7. $\displaystyle\sum_{n=2}^{\infty} \frac{(x-7)^n}{n\ln(n)}$
8. $\displaystyle\sum_{n=0}^{\infty} \frac{n!(x+1)^n}{n^n}$
9. $\displaystyle\sum_{n=1}^{\infty} \frac{(2x-3)^{2n}}{n4^n}$
10. $\displaystyle\sum_{n=1}^{\infty} \frac{n(x-5)^n}{2^n(n+1)}$